2.5 Prime Factorization and the Least Common Multiple

Learning Objectives

By the end of this section, you will be able to:

- Find the prime factorization of a composite number
- Find the least common multiple (LCM) of two numbers

Be Prepared!

Before you get started, take this readiness quiz.

1. Is 810 divisible by $2, 3, 5, 6,$ or 10?
   If you missed this problem, review **Find Multiples and Factors**.
2. Is 127 prime or composite?
   If you missed this problem, review **Find Multiples and Factors**.
3. Write $2 \cdot 2 \cdot 2 \cdot 2$ in exponential notation.
   If you missed this problem, review **Use the Language of Algebra**.

Find the Prime Factorization of a Composite Number

In the previous section, we found the factors of a number. Prime numbers have only two factors, the number 1 and the prime number itself. Composite numbers have more than two factors, and every composite number can be written as a unique product of primes. This is called the **prime factorization** of a number. When we write the prime factorization of a number, we are rewriting the number as a product of primes. Finding the prime factorization of a composite number will help you later in this course.

Prime Factorization

The prime factorization of a number is the product of prime numbers that equals the number.

MANIPULATIVE MATHEMATICS

Doing the Manipulative Mathematics activity “Prime Numbers” will help you develop a better sense of prime numbers.

You may want to refer to the following list of prime numbers less than 50 as you work through this section.

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$$

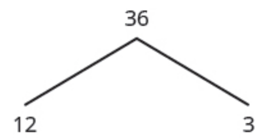
Prime Factorization Using the Factor Tree Method

One way to find the prime factorization of a number is to make a factor tree. We start by writing the number, and then writing it as the product of two factors. We write the factors below the number and connect them to the number with a small line segment—a “branch” of the factor tree.

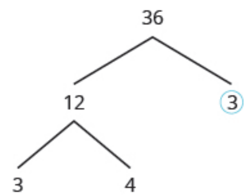
If a factor is prime, we circle it (like a bud on a tree), and do not factor that “branch” any further. If a factor is not prime, we repeat this process, writing it as the product of two factors and adding new branches to the tree.

We continue until all the branches end with a prime. When the factor tree is complete, the circled primes give us the prime factorization.

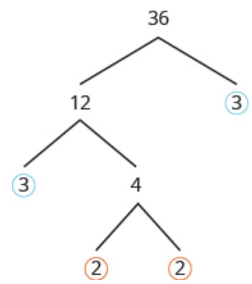
For example, let’s find the prime factorization of $36.$ We can start with any factor pair such as 3 and $12.$ We write 3 and 12 below 36 with branches connecting them.



The factor 3 is prime, so we circle it. The factor 12 is composite, so we need to find its factors. Let's use 3 and $4.$ We write these factors on the tree under the $12.$



The factor 3 is prime, so we circle it. The factor 4 is composite, and it factors into $2 \cdot 2.$ We write these factors under the $4.$ Since 2 is prime, we circle both $2\text{s}.$



The prime factorization is the product of the circled primes. We generally write the prime factorization in order from least to greatest.

$$2 \cdot 2 \cdot 3 \cdot 3$$

In cases like this, where some of the prime factors are repeated, we can write prime factorization in exponential form.

$$2 \cdot 2 \cdot 3 \cdot 3$$

$$2^2 \cdot 3^2$$

Note that we could have started our factor tree with any factor pair of $36.$ We chose 12 and $3,$ but the same result would have been the same if we had started with 2 and $18,$ 4 and $9,$ or 6 and $6.$

? **HOW TO : :** FIND THE PRIME FACTORIZATION OF A COMPOSITE NUMBER USING THE TREE METHOD.

Step 1. Find any factor pair of the given number, and use these numbers to create two branches.

Step 2. If a factor is prime, that branch is complete. Circle the prime.

Step 3. If a factor is not prime, write it as the product of a factor pair and continue the process.

Step 4. Write the composite number as the product of all the circled primes.

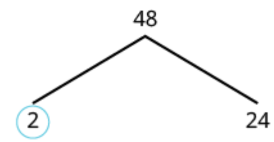
EXAMPLE 2.48

Find the prime factorization of 48 using the factor tree method.

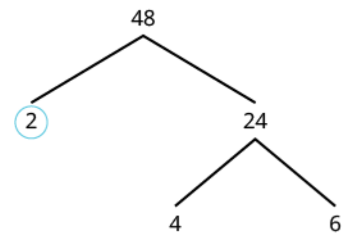
Solution

We can start our tree using any factor pair of 48. Let's use 2 and 24.

We circle the 2 because it is prime and so that branch is complete.

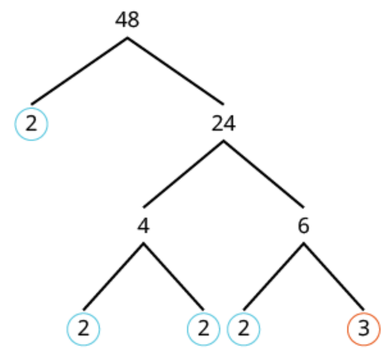


Now we will factor 24. Let's use 4 and 6.



Neither factor is prime, so we do not circle either.
We factor the 4, using 2 and 2.
We factor 6, using 2 and 3.

We circle the 2s and the 3 since they are prime. Now all of the branches end in a prime.



Write the product of the circled numbers.	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
Write in exponential form.	$2^4 \cdot 3$

Check this on your own by multiplying all the factors together. The result should be 48.

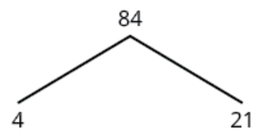
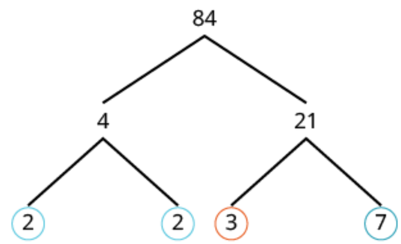
TRY IT : : 2.95 Find the prime factorization using the factor tree method: 80

TRY IT : : 2.96 Find the prime factorization using the factor tree method: 60

EXAMPLE 2.49

Find the prime factorization of 84 using the factor tree method.

Solution

We start with the factor pair 4 and 21. Neither factor is prime so we factor them further.	
Now the factors are all prime, so we circle them.	
Then we write 84 as the product of all circled primes.	$2 \cdot 2 \cdot 3 \cdot 7$ $2^2 \cdot 3 \cdot 7$

Draw a factor tree of 84.

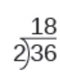
TRY IT : : 2.97 Find the prime factorization using the factor tree method: 126

TRY IT : : 2.98 Find the prime factorization using the factor tree method: 294

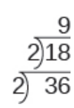
Prime Factorization Using the Ladder Method

The ladder method is another way to find the prime factors of a composite number. It leads to the same result as the factor tree method. Some people prefer the ladder method to the factor tree method, and vice versa.

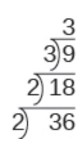
To begin building the "ladder," divide the given number by its smallest prime factor. For example, to start the ladder for $36,$ we divide 36 by $2,$ the smallest prime factor of $36.$



To add a "step" to the ladder, we continue dividing by the same prime until it no longer divides evenly.



Then we divide by the next prime; so we divide 9 by $3.$



We continue dividing up the ladder in this way until the quotient is prime. Since the quotient, $3,$ is prime, we stop here.

Do you see why the ladder method is sometimes called stacked division?

The prime factorization is the product of all the primes on the sides and top of the ladder.

$$2 \cdot 2 \cdot 3 \cdot 3$$

$$2^2 \cdot 3^2$$

Notice that the result is the same as we obtained with the factor tree method.

HOW TO : : FIND THE PRIME FACTORIZATION OF A COMPOSITE NUMBER USING THE LADDER METHOD.

Step 1. Divide the number by the smallest prime.

Step 2. Continue dividing by that prime until it no longer divides evenly.

Step 3. Divide by the next prime until it no longer divides evenly.

Step 4. Continue until the quotient is a prime.

Step 5. Write the composite number as the product of all the primes on the sides and top of the ladder.

EXAMPLE 2.50

Find the prime factorization of 120 using the ladder method.

Solution

Divide the number by the smallest prime, which is 2.	$\begin{array}{r} 60 \\ 2\overline{)120} \end{array}$
Continue dividing by 2 until it no longer divides evenly.	$\begin{array}{r} 5 \\ 3\overline{)15} \\ 2\overline{)\ 30} \\ 2\overline{)\ \ 60} \\ 2\overline{)\ \ \ 120} \end{array}$
Divide by the next prime, 3.	$\begin{array}{r} 15 \\ 2\overline{)30} \\ 2\overline{)\ 60} \\ 2\overline{)\ \ 120} \end{array}$
The quotient, 5, is prime, so the ladder is complete. Write the prime factorization of 120.	$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ $2^3 \cdot 3 \cdot 5$

Check this yourself by multiplying the factors. The result should be $120.$

TRY IT : : 2.99 Find the prime factorization using the ladder method: 80

TRY IT : : 2.100 Find the prime factorization using the ladder method: 60

EXAMPLE 2.51

Find the prime factorization of 48 using the ladder method.

Solution

Divide the number by the smallest prime, 2.	$\begin{array}{r} 24 \\ 2\overline{)48} \end{array}$
Continue dividing by 2 until it no longer divides evenly.	$\begin{array}{r} 3 \\ 2\overline{)6} \\ 2\overline{)\ 12} \\ 2\overline{)\ \ 24} \\ 2\overline{)\ \ \ 48} \end{array}$
The quotient, 3, is prime, so the ladder is complete. Write the prime factorization of 48.	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ $2^4 \cdot 3$

TRY IT : : 2.101 Find the prime factorization using the ladder method. 126

TRY IT : : 2.102 Find the prime factorization using the ladder method. 294

Find the Least Common Multiple (LCM) of Two Numbers

One of the reasons we look at multiples and primes is to use these techniques to find the least common multiple of two numbers. This will be useful when we add and subtract fractions with different denominators.

Listing Multiples Method

A common multiple of two numbers is a number that is a multiple of both numbers. Suppose we want to find common multiples of 10 and $25.$ We can list the first several multiples of each number. Then we look for multiples that are common to both lists—these are the common multiples.

$$\begin{array}{l} 10{:}10,\ 20,\ 30,\ 40,\ \mathbf{50},\ 60,\ 70,\ 80,\ 90,\ \mathbf{100},\ 110,\ \ldots \\ 25{:}25,\ \mathbf{50},\ 75,\ \mathbf{100},\ 125,\ \ldots \end{array}$$

We see that 50 and 100 appear in both lists. They are common multiples of 10 and $25.$ We would find more common multiples if we continued the list of multiples for each.

The smallest number that is a multiple of two numbers is called the **least common multiple** (LCM). So the least LCM of 10 and 25 is $50.$

HOW TO : : FIND THE LEAST COMMON MULTIPLE (LCM) OF TWO NUMBERS BY LISTING MULTIPLES.

Step 1. List the first several multiples of each number.

Step 2. Look for multiples common to both lists. If there are no common multiples in the lists, write out additional multiples for each number.

Step 3. Look for the smallest number that is common to both lists.

Step 4. This number is the LCM.

EXAMPLE 2.52

Find the LCM of 15 and 20 by listing multiples.

Solution

List the first several multiples of 15 and of $20.$ Identify the first common multiple.

$$\begin{array}{l} 15{:}\ 15,\ 30,\ 45,\ \mathbf{60},\ 75,\ 90,\ 105,\ 120 \\ 20{:}\ 20,\ 40,\ \mathbf{60},\ 80,\ 100,\ 120,\ 140,\ 160 \end{array}$$

The smallest number to appear on both lists is $60,$ so 60 is the least common multiple of 15 and $20.$

Notice that 120 is on both lists, too. It is a common multiple, but it is not the least common multiple.

TRY IT : : 2.103 Find the least common multiple (LCM) of the given numbers: $9 \text{ and } 12$

TRY IT : : 2.104 Find the least common multiple (LCM) of the given numbers: $18 \text{ and } 24$

Prime Factors Method

Another way to find the least common multiple of two numbers is to use their prime factors. We'll use this method to find the LCM of 12 and $18.$

We start by finding the prime factorization of each number.

$$12 = 2 \cdot 2 \cdot 3 \qquad 18 = 2 \cdot 3 \cdot 3$$

Then we write each number as a product of primes, matching primes vertically when possible.

$$\begin{array}{l} 12 = 2 \cdot 2 \cdot 3 \\ 18 = 2 \cdot \phantom{2 \cdot{}} 3 \cdot 3 \end{array}$$

Now we bring down the primes in each column. The LCM is the product of these factors.

$$\begin{array}{rl} 12 = & 2 \cdot 2 \cdot 3 \\ 18 = & 2 \cdot \quad\;\; 3 \cdot 3 \\ \hline \text{LCM} = & 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCM} = & 2 \cdot 2 \cdot 3 \cdot 3 = 36 \end{array}$$

Notice that the prime factors of 12 and the prime factors of 18 are included in the LCM. By matching up the common primes, each common prime factor is used only once. This ensures that 36 is the least common multiple.

HOW TO : : FIND THE LCM USING THE PRIME FACTORS METHOD.

Step 1. Find the prime factorization of each number.

Step 2. Write each number as a product of primes, matching primes vertically when possible.

Step 3. Bring down the primes in each column.

Step 4. Multiply the factors to get the LCM.

EXAMPLE 2.53

Find the LCM of 15 and 18 using the prime factors method.

Solution

Write each number as a product of primes.	$15 = 3 \cdot 5 \qquad 18 = 2 \cdot 3 \cdot 3$
Write each number as a product of primes, matching primes vertically when possible.	$\begin{array}{rl} 15 = & \quad\;\; 3 \cdot \quad\;\; 5 \\ 18 = & 2 \cdot 3 \cdot 3 \end{array}$
Bring down the primes in each column.	$\begin{array}{rl} 15 = & \quad\;\; 3 \cdot \quad\;\; 5 \\ 18 = & 2 \cdot 3 \cdot 3 \\ \hline \text{LCM} = & 2 \cdot 3 \cdot 3 \cdot 5 \end{array}$
Multiply the factors to get the LCM.	$\text{LCM} = 2 \cdot 3 \cdot 3 \cdot 5$ The LCM of 15 and 18 is 90.

TRY IT : : 2.105 Find the LCM using the prime factors method. 15 and 20

TRY IT : : 2.106 Find the LCM using the prime factors method. 15 and 35

EXAMPLE 2.54

Find the LCM of 50 and 100 using the prime factors method.

Solution

Write the prime factorization of each number.	$50 = 2 \cdot 5 \cdot 5 \qquad 100 = 2 \cdot 2 \cdot 5 \cdot 5$
Write each number as a product of primes, matching primes vertically when possible.	$\begin{array}{rl} 50 = & \quad\;\; 2 \cdot 5 \cdot 5 \\ 100 = & 2 \cdot 2 \cdot 5 \cdot 5 \end{array}$
Bring down the primes in each column.	$\begin{array}{rl} 50 = & \quad\;\; 2 \cdot 5 \cdot 5 \\ 100 = & 2 \cdot 2 \cdot 5 \cdot 5 \\ \hline \text{LCM} = & 2 \cdot 2 \cdot 5 \cdot 5 \end{array}$
Multiply the factors to get the LCM.	$\text{LCM} = 2 \cdot 2 \cdot 5 \cdot 5$ The LCM of 50 and 100 is 100.

TRY IT : : 2.107 Find the LCM using the prime factors method: $55, 88$

TRY IT : : 2.108 Find the LCM using the prime factors method: $60, 72$

MEDIA : : ACCESS ADDITIONAL ONLINE RESOURCES

- Ex 1: Prime Factorization (http://openstaxcollege.org/l/24PrimeFactor1)
- Ex 2: Prime Factorization (http://openstaxcollege.org/l/24PrimeFactor2)
- Ex 3: Prime Factorization (http://openstaxcollege.org/l/24PrimeFactor3)
- Ex 1: Prime Factorization Using Stacked Division (http://openstaxcollege.org/l/24stackeddivis)
- Ex 2: Prime Factorization Using Stacked Division (http://openstaxcollege.org/l/24stackeddivis2)
- The Least Common Multiple (http://openstaxcollege.org/l/24LCM)
- Example: Determining the Least Common Multiple Using a List of Multiples (http://openstaxcollege.org/l/24LCM2)
- Example: Determining the Least Common Multiple Using Prime Factorization (http://openstaxcollege.org/l/24LCMFactor)