

4.2 Multiply and Divide Fractions

By the end of this section, you will be able to:

- Simplify fractions
- Multiply fractions
- Find reciprocals
- Divide fractions

Simplify Fractions

In working with equivalent fractions, you saw that there are many ways to write fractions that have the same value, or represent the same part of the whole. How do you know which one to use? Often, we'll use the fraction that is in *simplified* form.

A fraction is considered simplified if there are no common factors, other than 1, in the **numerator** and **denominator**. If a fraction does have common factors in the numerator and denominator, we can reduce the fraction to its simplified form by removing the common factors.

SIMPLIFIED FRACTION

A fraction is considered simplified if there are no common factors in the numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

The process of simplifying a fraction is often called *reducing the fraction*. In the previous section, we used the Equivalent Fractions Property to find equivalent fractions. We can also use the Equivalent

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Fractions Property in reverse to simplify fractions. We rewrite the property to show both forms together.

NOTE: EQUIVALENT FRACTIONS PROPERTY

If a, b, c are numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}.$$

Notice that c is a common factor in the **numerator** and **denominator**. Anytime we have a common factor in the numerator and denominator, it can be removed.

SIMPLIFY A FRACTION.

1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
2. Simplify, using the equivalent fractions property, by removing common factors.
3. Multiply any remaining factors.

Simplify: $\frac{10}{15}$.

Solution

To simplify the fraction, we look for any common factors in the numerator and the denominator.

Notice that 5 is a factor of both 10 and 15.

$$\frac{10}{15}$$

Factor the numerator and denominator.

$$\frac{2 \cdot 5}{3 \cdot 5}$$

Remove the common factors.

$$\frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}}$$

Simplify.

$$\frac{2}{3}$$

Simplify: $\frac{8}{12}$.

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$$\frac{2}{3}$$

Simplify: $\frac{12}{16}$.

$$\frac{3}{4}$$

To simplify a negative fraction, we use the same process. Remember to keep the negative sign.

Simplify: $-\frac{18}{24}$.

Solution

We notice that 18 and 24 both have factors

$$-\frac{18}{24}$$

Rewrite the numerator and denominator showing the common factor

$$-\frac{3 \cdot 6}{4 \cdot 6}$$

Remove common factors

$$-\frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}}$$

Simplify.

$$-\frac{3}{4}$$

Simplify: $-\frac{21}{28}$.

$$-\frac{3}{4}$$

Simplify: $-\frac{16}{24}$.

$$-\frac{2}{3}$$

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After simplifying a fraction, it is always important to check the result to make sure that the numerator and denominator do not have any more factors in common. Remember, the definition of a simplified fraction: *a fraction is considered simplified if there are no common factors in the numerator and denominator.*

When we simplify an improper fraction, there is no need to change it to a mixed number.

Simplify: $-\frac{56}{32}$.

Solution

$$-\frac{56}{32}$$

Rewrite the numerator and denominator, showing the common factors, 8.

$$\frac{7 \cdot 8}{4 \cdot 8}$$

Remove common factors.

$$\frac{7 \cdot \cancel{8}}{4 \cdot \cancel{8}}$$

Simplify.

$$-\frac{7}{4}$$

So, the simplified form of $-\frac{56}{32}$ is $-\frac{7}{4}$.

Simplify: $-\frac{54}{42}$.

$$-\frac{9}{7}$$

Simplify: $-\frac{81}{45}$.

$$-\frac{9}{5}$$

SIMPLIFY A FRACTION.

1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
2. Simplify, using the equivalent fractions property, by removing common factors.

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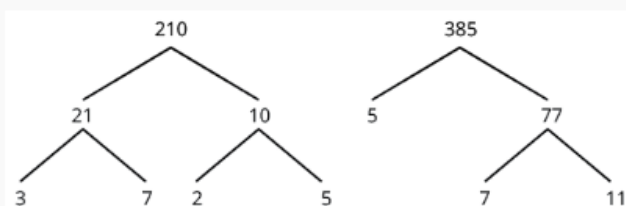
3. Multiply any remaining factors

Sometimes it may not be easy to find common factors of the numerator and denominator. A good idea, then, is to factor the numerator and the denominator into prime numbers. (You may want to use the factor tree method to identify the prime factors.) Then divide out the common factors using the Equivalent Fractions Property.

Simplify: $\frac{210}{385}$.

Use factor trees to factor the numerator and denominator.

$$\frac{210}{385}$$



Rewrite the numerator and denominator as the product of the primes.

$$\frac{210}{385} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{5 \cdot 7 \cdot 11}$$

Remove the common factors.

$$\frac{2 \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}{\cancel{5} \cdot \cancel{7} \cdot 11}$$

Simplify.

$$\frac{2 \cdot 3}{11}$$

Multiply any remaining factors.

$$\frac{6}{11}$$

Simplify: $\frac{69}{120}$.

$$\frac{23}{40}$$

Simplify: $\frac{120}{192}$.

$$\frac{5}{8}$$

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We can also simplify fractions containing variables. If a variable is a common factor in the **numerator** and **denominator**, we remove it just as we do with an integer factor.

Simplify: $\frac{5xy}{15x}$.

Solution

	$\frac{5xy}{15x}$
Rewrite numerator and denominator showing common factors.	$\frac{5 \cdot x \cdot y}{3 \cdot 5 \cdot x}$
Remove common factors.	$\frac{\cancel{5} \cdot \cancel{x} \cdot y}{3 \cdot \cancel{5} \cdot \cancel{x}}$
Simplify.	$\frac{y}{3}$

Simplify: $\frac{7x}{7y}$.

$$\frac{x}{y}$$

Simplify: $\frac{9a}{9b}$.

$$\frac{a}{b}$$

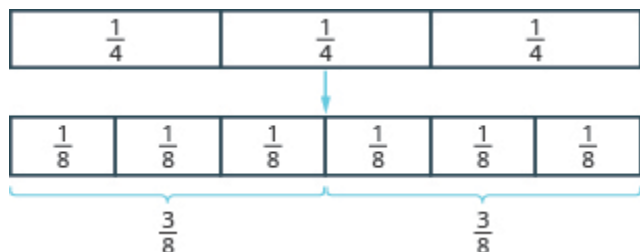
Multiply fractions

A model may help you understand multiplication of fractions. We will use fraction tiles to model $\frac{1}{2} \cdot \frac{3}{4}$. To multiply $\frac{1}{2}$ and $\frac{3}{4}$, think $\frac{1}{2}$ of $\frac{3}{4}$.

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Start with fraction tiles for three-fourths. To find one-half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three $\frac{1}{4}$ tiles evenly into two parts, we exchange them for smaller tiles.



We see $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Taking half of the six $\frac{1}{8}$ tiles gives us three $\frac{1}{8}$ tiles, which is $\frac{3}{8}$.

Therefore,

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

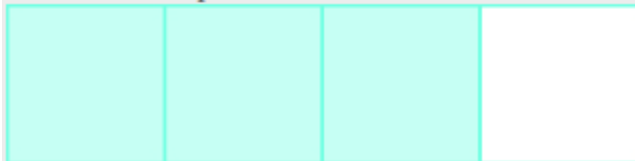
Doing the Manipulative Mathematics activity Model Fraction Multiplication will help you develop a better understanding of how to multiply fractions.

Use a diagram to model $\frac{1}{2} \cdot \frac{3}{4}$.

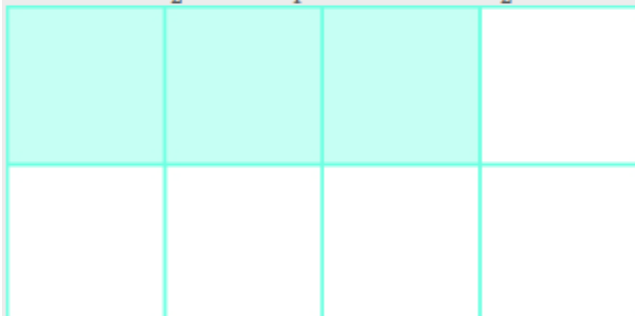
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Solution

First shade in $\frac{3}{4}$ of the rectangle.



We will take $\frac{1}{2}$ of this $\frac{3}{4}$, so we shade $\frac{1}{2}$ of the shaded region.



Notice that 3 out of the 8 pieces are shaded. This means that $\frac{3}{8}$ of the rectangle is shaded.

Therefore, $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$, or $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

Use a diagram to model: $\frac{1}{2} \cdot \frac{3}{5}$.

$$\frac{3}{10}$$

Use a diagram to model: $\frac{1}{2} \cdot \frac{5}{6}$.

$$\frac{5}{12}$$

Earlier we found that $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$. Do you notice that we could have gotten the same answer by multiplying the numerators and multiplying the denominators?

$$\frac{1}{2} \cdot \frac{3}{4}$$

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Multiply the numerators, and multiply the denominators

$$\frac{1}{2} \cdot \frac{3}{4}$$

Simplify

$$\frac{3}{8}$$

This leads to the definition of fraction multiplication. To multiply fractions, we multiply the numerators and multiply the denominators. Then we write the fraction in simplified form.

NOTE: FRACTION MULTIPLICATION

If $a, b, c,$ and d are numbers where $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. After multiplying, we write the fraction in simplified form. We will multiply two negatives, so the product will be positive.

Multiply, and write the answer in simplified form: $\frac{3}{4} \cdot \frac{1}{5}$.

Solution

	$\frac{3}{4} \cdot \frac{1}{5}$
Multiply the numerators; multiply the denominators	$\frac{3 \cdot 1}{4 \cdot 5}$
Simplify	$\frac{3}{20}$

There are no common factors, so the fraction is simplified.

Multiply, and write the answer in simplified form: $\frac{1}{3} \cdot \frac{2}{5}$.

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$$\frac{2}{15}$$

Multiply, and write the answer in simplified form: $\frac{3}{5} \cdot \frac{7}{8}$.

$$\frac{21}{40}$$

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. We will multiply two negatives, so the product will be positive.

Multiply, and write the answer in simplified form: $-\frac{5}{8} \left(-\frac{2}{3}\right)$.

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Solution

$$-\frac{5}{8}\left(-\frac{2}{3}\right)$$

The signs are the same, so the product is positive. Multiply the numerators, multiply the denominators.

$$\frac{5 \cdot 2}{8 \cdot 3}$$

Simplify.

$$\frac{10}{24}$$

Look for common factors in the numerator and denominator. Rewrite showing common factors.

$$\frac{5 \cdot 2}{12 \cdot 2}$$

Remove common factors.

$$\frac{5}{12}$$

Another way to find this product involves removing common factors earlier.

$$-\frac{5}{8}\left(-\frac{2}{3}\right)$$

Determine the sign of the product. Multiply.

$$\frac{5 \cdot 2}{8 \cdot 3}$$

Show common factors and then remove them.

$$\frac{5 \cdot \cancel{2}}{4 \cdot \cancel{2} \cdot 3}$$

Multiply remaining factors.

$$\frac{5}{12}$$

We get the same result.

Multiply, and write the answer in simplified form: $-\frac{4}{7}\left(-\frac{5}{8}\right)$.

$$\frac{5}{14}$$

Multiply, and write the answer in simplified form: $-\frac{7}{12}\left(-\frac{8}{9}\right)$.

$$\frac{14}{27}$$

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Multiply, and write the answer in simplified form: $-\frac{14}{15} \cdot \frac{20}{21}$.

Solution

$$-\frac{14}{15} \cdot \frac{20}{21}$$

Determine the sign of the product; multiply.

$$-\frac{14}{15} \cdot \frac{20}{21}$$

Are there any common factors in the numerator and the denominator?

We know that 7 is a factor of 14 and 21, and 5 is a factor of 20 and 15.

Rewrite showing common factors.

$$-\frac{2 \cdot \cancel{7} \cdot 4 \cdot \cancel{5}}{3 \cdot \cancel{5} \cdot 3 \cdot \cancel{7}}$$

Remove the common factors.

$$-\frac{2 \cdot 4}{3 \cdot 3}$$

Multiply the remaining factors.

$$-\frac{8}{9}$$

Multiply, and write the answer in simplified form: $-\frac{10}{28} \cdot \frac{8}{15}$.

$$-\frac{4}{21}$$

Multiply, and write the answer in simplified form: $-\frac{9}{20} \cdot \frac{5}{12}$.

$$-\frac{3}{16}$$

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When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a , can be written as $a\frac{1}{1}$. So, $3 = \frac{3}{1}$, for example.

Multiply, and write the answer in simplified form:

Ⓐ $\frac{1}{7} \cdot 56$

Ⓑ $\frac{12}{5} (-20x)$

Solution

Ⓐ

$$\frac{1}{7} \cdot 56$$

Write 56 as a fraction.

$$\frac{1}{7} \cdot \frac{56}{1}$$

Determine the sign of the product; multiply.

$$\frac{56}{7}$$

Simplify.

$$8$$

Ⓑ

$$\frac{12}{5} (-20x)$$

Write $-20x$ as a fraction.

$$\frac{12}{5} \left(\frac{-20x}{1} \right)$$

Determine the sign of the product; multiply.

$$-\frac{12 \cdot 20x}{5 \cdot 1}$$

Show common factors and then remove them.

$$-\frac{12 \cdot \cancel{4} \cdot \cancel{5}x}{\cancel{5} \cdot 1}$$

Multiply remaining factors; simplify.

$$-48x$$

Multiply, and write the answer in simplified form:

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Ⓐ $\frac{1}{8} \cdot 72$ Ⓑ $\frac{11}{3}(-9a)$

1. Ⓐ 9

2. Ⓑ $-33a$

Multiply, and write the answer in simplified form:

Ⓐ $\frac{3}{8} \cdot 64$

Ⓑ $16x \cdot \frac{11}{12}$

Find Reciprocals

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are related to each other in a special way. So

are $-\frac{10}{7}$ and $-\frac{7}{10}$.

Do you see how? Besides looking like upside-down versions of one another, if we were to multiply these pairs of fractions, the product would be 1.

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \text{ and } -\frac{10}{7} \left(-\frac{7}{10}\right) = 1$$

Such pairs of numbers are called reciprocals.

RECIPROCAL

The **reciprocal** of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, where $a \neq 0$ and $b \neq 0$.

A number and its reciprocal have a product of 1.

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

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To find the reciprocal of a fraction, we invert the fraction. This means that we place the numerator in the denominator and the denominator in the numerator.

To get a positive result when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

$$\frac{a}{b} \cdot \frac{b}{a} = 1 \text{ positive}$$

$$3 \cdot \frac{1}{3} = 1 \quad \text{and} \quad -3 \cdot \left(-\frac{1}{3}\right) = 1$$

both positive

both negative

To find the reciprocal, keep the same sign and invert the fraction. The number zero does not have a reciprocal. Why? A number and its reciprocal multiply to 1. Is there any number r so that

$0 \cdot r = 1$? No. So, the number 0 does not have a reciprocal.

Find the reciprocal of each number. Then check that the product of each number and its reciprocal is 1.

1. (a) $\frac{4}{9}$

2. (b) $-\frac{1}{6}$

3. (c) $-\frac{14}{5}$

4. (d) 7

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Solution

To find the reciprocals, we keep the sign and invert the fractions.

Ⓐ

Find the reciprocal of $\frac{4}{9}$.

The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$.

Check:

Multiply the number and its reciprocal.

$$\frac{4}{9} \cdot \frac{9}{4}$$

Multiply numerators and denominators.

$$\frac{36}{36}$$

Simplify.

$$1\checkmark$$

Ⓑ

Find the reciprocal of $-\frac{1}{6}$.

$$-\frac{6}{1}$$

Simplify.

$$-6$$

Check:

$$-\frac{1}{6} \cdot (-6)$$

$$1\checkmark$$

Ⓒ

Find the reciprocal of $-\frac{14}{5}$.

$$-\frac{5}{14}$$

Check:

$$-\frac{14}{5} \cdot \left(-\frac{5}{14}\right)$$

$$\frac{70}{70}$$

$$1\checkmark$$

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④

Find the reciprocal of 7.

Write 7 as a fraction.

$$\frac{7}{1}$$

Write the reciprocal of $\frac{7}{1}$.

$$\frac{1}{7}$$

Check:

$$7 \cdot \left(\frac{1}{7}\right)$$

$$1\checkmark$$

Find the reciprocal:

Ⓐ $\frac{5}{7}$

Ⓑ $-\frac{1}{8}$

Ⓒ $-\frac{11}{4}$

Ⓓ 14

1. Ⓐ $\frac{7}{5}$

2. Ⓑ -8

3. Ⓒ $-\frac{4}{11}$

4. Ⓓ $\frac{1}{14}$

Find the reciprocal:

Ⓐ $\frac{3}{7}$

Ⓑ $-\frac{1}{12}$

Ⓒ $-\frac{14}{9}$

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Ⓓ 21

1. Ⓐ $\frac{7}{3}$
2. Ⓑ -12
3. Ⓒ $-\frac{9}{14}$
4. Ⓓ $\frac{1}{21}$

In a previous chapter, we worked with opposites and absolute values. The table below compares opposites, absolute values, and reciprocals.

Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

Fill in the chart for each fraction in the left column:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$			
$\frac{1}{2}$			
$\frac{9}{5}$			
-5			

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Solution

To find the opposite, change the sign. To find the absolute value, leave the positive numbers the same, but take the opposite of the negative numbers. To find the reciprocal, keep the sign the same and **invert** the fraction.

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$-\frac{8}{3}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2
$\frac{9}{5}$	$-\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$
-5	5	5	$-\frac{1}{5}$

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$			
$\frac{1}{4}$			
$\frac{8}{3}$			
-8			

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$-\frac{8}{5}$
$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	4
$\frac{8}{3}$	$-\frac{8}{3}$	$\frac{8}{3}$	$\frac{3}{8}$
-8	8	8	$-\frac{1}{8}$

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Divide Fractions

Why is $12 \div 3 = 4$? We previously modeled this with counters. How many groups of 3 counters can be made from a group of 12 counters?



There are 4 groups of 3 counters. In other words, there are four 3s in 12. So, $12 \div 3 = 4$.

What about dividing fractions? Suppose we want to find the quotient: $\frac{1}{2} \div \frac{1}{6}$.

We need to figure out how many $\frac{1}{6}$ s there are in $\frac{1}{2}$.

We can use fraction tiles to model this division. We start by lining up the half and sixth fraction tiles as shown below. Notice, there are three $\frac{1}{6}$ tiles in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{6} = \frac{1}{3}$.

$\frac{1}{2}$		
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Doing the Manipulative Mathematics activity Model Fraction Division will help you develop a better understanding of dividing fractions.

Model: $\frac{1}{4} \div \frac{1}{8}$.

Solution

We want to determine how many $\frac{1}{8}$ s are in $\frac{1}{4}$. Start with one $\frac{1}{4}$ tile. Line up $\frac{1}{8}$ tiles underneath the $\frac{1}{4}$ tile.

$\frac{1}{4}$	
$\frac{1}{8}$	$\frac{1}{8}$

There are two $\frac{1}{8}$ s in $\frac{1}{4}$.

So, $\frac{1}{4} \div \frac{1}{8} = 2$.

[4.2 Multiply and Divide Fractions](http://cnx.org/contents/yqV9q0HH@9.6:s7ku6WX5@3/Multiply-and-Divide-Fractions) from Prealgebra by OpenStax is available under a [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. © Jul 20, 2016, OpenStax.

Model: $\frac{1}{3} \div \frac{1}{6}$.

$\frac{1}{3}$	
$\frac{1}{6}$	$\frac{1}{6}$

Model: $\frac{1}{2} \div \frac{1}{4}$.

$\frac{1}{2}$	
$\frac{1}{4}$	$\frac{1}{4}$

Model: $2 \div \frac{1}{4}$.

Solution

We are trying to determine how many $\frac{1}{4}$ s there are in 2. We can model this as shown.

1				1			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Because there are eight $\frac{1}{4}$ s in 2, $2 \div \frac{1}{4} = 8$.

Model: $2 \div \frac{1}{3}$.

1			1		
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Model: $3 \div \frac{1}{2}$.

1		1		1	
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Let's use money to model $2 \div \frac{1}{4}$ in another way. We often read $\frac{1}{4}$ as a 'quarter', and we know that a

quarter is one-fourth of a dollar as shown in the figure below. So we can think of $2 \div \frac{1}{4}$

as, "How many quarters are there in two dollars?" One dollar is 4 quarters, so 2dollars would be 8 quarters. So again, $2 \div \frac{1}{4} = 8$.

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The U.S. coin called a quarter is worth one-fourth of a dollar.

Using fraction tiles, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \cdot \frac{6}{1} = 3$

also. How are $\frac{1}{6}$ and $\frac{6}{1}$ related? They are reciprocals. This leads us to the procedure for fraction division.

NOTE: FRACTION DIVISION

If a, b, c , and d are numbers where $b \neq 0, c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide fractions, multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0, c \neq 0$, and $d \neq 0$ to be sure we don't divide by zero.

Divide, and write the answer in simplified form: $\frac{2}{5} \div \left(-\frac{3}{7}\right)$.

Solution

$$\frac{2}{5} \div \left(-\frac{3}{7}\right)$$

Multiply the first fraction by the reciprocal of the second.

$$\frac{2}{5} \left(-\frac{7}{3}\right)$$

Multiply. The product is negative.

$$-\frac{14}{15}$$

Divide, and write the answer in simplified form: $\frac{3}{7} \div \left(-\frac{2}{3}\right)$.

$$-\frac{9}{14}$$

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Divide, and write the answer in simplified form: $\frac{2}{3} \div (-\frac{7}{5})$.

$$-\frac{10}{21}$$

Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{n}{5}$.

Solution

$$\frac{2}{3} \div \frac{n}{5}$$

Multiply the first fraction by the reciprocal of the second.

$$\frac{2}{3} \cdot \frac{5}{n}$$

Multiply.

$$\frac{10}{3n}$$

Divide, and write the answer in simplified form: $\frac{3}{5} \div \frac{p}{7}$.

$$\frac{21}{5p}$$

Divide, and write the answer in simplified form: $\frac{5}{8} \div \frac{q}{3}$.

$$\frac{15}{8q}$$

Divide, and write the answer in simplified form: $-\frac{3}{4} \div (-\frac{7}{8})$.

[4.2 Multiply and Divide Fractions](http://cnx.org/contents/yqV9q0HH@9.6:s7ku6WX5@3/Multiply-and-Divide-Fractions) from Prealgebra by OpenStax is available under a [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. © Jul 20, 2016, OpenStax.

Solution

	$-\frac{3}{4} \div \left(-\frac{7}{8}\right)$
Multiply the first fraction by the reciprocal of the second.	$-\frac{3}{4} \cdot \left(-\frac{8}{7}\right)$
Multiply. Remember to determine the sign first.	$\frac{3 \cdot 8}{4 \cdot 7}$
Rewrite to show common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 7}$
Remove common factors and simplify.	$\frac{6}{7}$

Divide, and write the answer in simplified form: $-\frac{2}{3} \div \left(-\frac{5}{6}\right)$.

$$\frac{4}{5}$$

Divide, and write the answer in simplified form: $-\frac{5}{6} \div \left(-\frac{2}{3}\right)$.

$$\frac{5}{4}$$

Divide, and write the answer in simplified form: $\frac{7}{18} \div \frac{14}{27}$.

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Solution

	$\frac{7}{18} \div \frac{14}{27}$
Multiply the first fraction by the reciprocal of the second.	$\frac{7}{18} \cdot \frac{27}{14}$
Multiply.	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{\cancel{7} \cdot \cancel{9} \cdot 3}{\cancel{9} \cdot 2 \cdot \cancel{7} \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

Divide, and write the answer in simplified form: $\frac{7}{27} \div \frac{35}{36}$.

$$\frac{4}{15}$$

Divide, and write the answer in simplified form: $\frac{5}{14} \div \frac{15}{28}$.

$$\frac{2}{3}$$

Key Concepts

- **Equivalent Fractions Property**

◦ If a, b, c are numbers where $b \neq 0$, $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

- **Simplify a fraction.**

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1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
2. Simplify, using the equivalent fractions property, by removing common factors.
3. Multiply any remaining factors.

- **Fraction Multiplication**

- If a, b, c , and d are numbers where $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

- **Reciprocal**

- A number and its reciprocal have a product of 1. $\frac{a}{b} \cdot \frac{b}{a} = 1$

◦

Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

- **Fraction Division**

- If a, b, c , and d are numbers where $b \neq 0$, $c \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

- To divide fractions, multiply the first fraction by the reciprocal of the second.