Practice Test 1 from Introductory Statistics by OpenStax is available under a Creative Commons Attribution 4.0 International license. © June 23, 2016, OpenStax. UMGC has modified this work and it is available under the original license.

## 1 | SAMPLING AND DATA



Figure 1.1 We encounter statistics in our daily lives more often than we probably realize and from many different sources, like the news. (credit: David Sim)

## Introduction

## Chapter Objectives

By the end of this chapter, the student should be able to:

- Recognize and differentiate between key terms.
- Apply various types of sampling methods to data collection.
- Create and interpret frequency tables.

You are probably asking yourself the question, "When and where will I use statistics?" If you read any newspaper, watch television, or use the Internet, you will see statistical information. There are statistics about crime, sports, education, politics, and real estate. Typically, when you read a newspaper article or watch a television news program, you are given sample information. With this information, you may make a decision about the correctness of a statement, claim, or "fact." Statistical methods can help you make the "best educated guess."
Since you will undoubtedly be given statistical information at some point in your life, you need to know some techniques for analyzing the information thoughtfully. Think about buying a house or managing a budget. Think about your chosen profession. The fields of economics, business, psychology, education, biology, law, computer science, police science, and early childhood development require at least one course in statistics.
Included in this chapter are the basic ideas and words of probability and statistics. You will soon understand that statistics and probability work together. You will also learn how data are gathered and what "good" data can be distinguished from "bad."

## 1.1 | Definitions of Statistics, Probability, and Key Terms

The science of statistics deals with the collection, analysis, interpretation, and presentation of data. We see and use data in our everyday lives.

## ollaborative Exercise

In your classroom, try this exercise. Have class members write down the average time (in hours, to the nearest halfhour) they sleep per night. Your instructor will record the data. Then create a simple graph (called a dot plot) of the data. A dot plot consists of a number line and dots (or points) positioned above the number line. For example, consider the following data:
5; 5.5; 6; 6; 6; 6.5; 6.5; 6.5; 6.5; 7; 7; 8; 8; 9
The dot plot for this data would be as follows:

## Frequency of Average Time (in Hours) Spent Sleeping per Night



Figure 1.2

Does your dot plot look the same as or different from the example? Why? If you did the same example in an English class with the same number of students, do you think the results would be the same? Why or why not?

Where do your data appear to cluster? How might you interpret the clustering?
The questions above ask you to analyze and interpret your data. With this example, you have begun your study of statistics.

In this course, you will learn how to organize and summarize data. Organizing and summarizing data is called descriptive statistics. Two ways to summarize data are by graphing and by using numbers (for example, finding an average). After you have studied probability and probability distributions, you will use formal methods for drawing conclusions from "good" data. The formal methods are called inferential statistics. Statistical inference uses probability to determine how confident we can be that our conclusions are correct.
Effective interpretation of data (inference) is based on good procedures for producing data and thoughtful examination of the data. You will encounter what will seem to be too many mathematical formulas for interpreting data. The goal of statistics is not to perform numerous calculations using the formulas, but to gain an understanding of your data. The calculations can be done using a calculator or a computer. The understanding must come from you. If you can thoroughly grasp the basics of statistics, you can be more confident in the decisions you make in life.

## Probability

Probability is a mathematical tool used to study randomness. It deals with the chance (the likelihood) of an event occurring. For example, if you toss a fair coin four times, the outcomes may not be two heads and two tails. However, if you toss the same coin 4,000 times, the outcomes will be close to half heads and half tails. The expected theoretical probability of heads in any one toss is $\frac{1}{2}$ or 0.5 . Even though the outcomes of a few repetitions are uncertain, there is a regular pattern of outcomes when there are many repetitions. After reading about the English statistician Karl Pearson who tossed a coin 24,000 times with a result of 12,012 heads, one of the authors tossed a coin 2,000 times. The results were 996 heads. The fraction $\frac{996}{2000}$ is equal to 0.498 which is very close to 0.5 , the expected probability.

The theory of probability began with the study of games of chance such as poker. Predictions take the form of probabilities. To predict the likelihood of an earthquake, of rain, or whether you will get an A in this course, we use probabilities. Doctors use probability to determine the chance of a vaccination causing the disease the vaccination is supposed to prevent. A
stockbroker uses probability to determine the rate of return on a client's investments. You might use probability to decide to buy a lottery ticket or not. In your study of statistics, you will use the power of mathematics through probability calculations to analyze and interpret your data.

## Key Terms

In statistics, we generally want to study a population. You can think of a population as a collection of persons, things, or objects under study. To study the population, we select a sample. The idea of sampling is to select a portion (or subset) of the larger population and study that portion (the sample) to gain information about the population. Data are the result of sampling from a population.

Because it takes a lot of time and money to examine an entire population, sampling is a very practical technique. If you wished to compute the overall grade point average at your school, it would make sense to select a sample of students who attend the school. The data collected from the sample would be the students' grade point averages. In presidential elections, opinion poll samples of $1,000-2,000$ people are taken. The opinion poll is supposed to represent the views of the people in the entire country. Manufacturers of canned carbonated drinks take samples to determine if a 16 ounce can contains 16 ounces of carbonated drink.
From the sample data, we can calculate a statistic. A statistic is a number that represents a property of the sample. For example, if we consider one math class to be a sample of the population of all math classes, then the average number of points earned by students in that one math class at the end of the term is an example of a statistic. The statistic is an estimate of a population parameter. A parameter is a number that is a property of the population. Since we considered all math classes to be the population, then the average number of points earned per student over all the math classes is an example of a parameter.
One of the main concerns in the field of statistics is how accurately a statistic estimates a parameter. The accuracy really depends on how well the sample represents the population. The sample must contain the characteristics of the population in order to be a representative sample. We are interested in both the sample statistic and the population parameter in inferential statistics. In a later chapter, we will use the sample statistic to test the validity of the established population parameter.
A variable, notated by capital letters such as $X$ and $Y$, is a characteristic of interest for each person or thing in a population. Variables may be numerical or categorical. Numerical variables take on values with equal units such as weight in pounds and time in hours. Categorical variables place the person or thing into a category. If we let $X$ equal the number of points earned by one math student at the end of a term, then $X$ is a numerical variable. If we let $Y$ be a person's party affiliation, then some examples of $Y$ include Republican, Democrat, and Independent. $Y$ is a categorical variable. We could do some math with values of $X$ (calculate the average number of points earned, for example), but it makes no sense to do math with values of $Y$ (calculating an average party affiliation makes no sense).

Data are the actual values of the variable. They may be numbers or they may be words. Datum is a single value.
Two words that come up often in statistics are mean and proportion. If you were to take three exams in your math classes and obtain scores of 86,75 , and 92 , you would calculate your mean score by adding the three exam scores and dividing by three (your mean score would be 84.3 to one decimal place). If, in your math class, there are 40 students and 22 are men and 18 are women, then the proportion of men students is $\frac{22}{40}$ and the proportion of women students is $\frac{18}{40}$. Mean and proportion are discussed in more detail in later chapters.

## NOTE

The words " mean" and " average" are often used interchangeably. The substitution of one word for the other is common practice. The technical term is "arithmetic mean," and "average" is technically a center location. However, in practice among non-statisticians, "average" is commonly accepted for "arithmetic mean."

## Example 1.1

Determine what the key terms refer to in the following study. We want to know the average (mean) amount of money first year college students spend at ABC College on school supplies that do not include books. We randomly survey 100 first year students at the college. Three of those students spent $\$ 150, \$ 200$, and $\$ 225$, respectively.

## Solution 1.1

The population is all first year students attending ABC College this term.
The sample could be all students enrolled in one section of a beginning statistics course at ABC College (although this sample may not represent the entire population).

The parameter is the average (mean) amount of money spent (excluding books) by first year college students at ABC College this term.
The statistic is the average (mean) amount of money spent (excluding books) by first year college students in the sample.
The variable could be the amount of money spent (excluding books) by one first year student. Let $X=$ the amount of money spent (excluding books) by one first year student attending ABC College.
The data are the dollar amounts spent by the first year students. Examples of the data are $\$ 150$, $\$ 200$, and $\$ 225$.

## Try It $\Sigma \cdots$

1.1 Determine what the key terms refer to in the following study. We want to know the average (mean) amount of money spent on school uniforms each year by families with children at Knoll Academy. We randomly survey 100 families with children in the school. Three of the families spent $\$ 65, \$ 75$, and $\$ 95$, respectively.

## Example 1.2

Determine what the key terms refer to in the following study.
A study was conducted at a local college to analyze the average cumulative GPA's of students who graduated last year. Fill in the letter of the phrase that best describes each of the items below.

1. $\qquad$ Population 2. $\qquad$ Statistic 3. $\qquad$ Parameter 4. $\qquad$ Sample 5. $\qquad$ Variable 6. $\qquad$ Data
a) all students who attended the college last year
b) the cumulative GPA of one student who graduated from the college last year
c) $3.65,2.80,1.50,3.90$
d) a group of students who graduated from the college last year, randomly selected
e) the average cumulative GPA of students who graduated from the college last year
f) all students who graduated from the college last year
g) the average cumulative GPA of students in the study who graduated from the college last year

## Solution 1.2

1. f; 2. g; 3. e; 4. d; 5. b; 6. c

## Example 1.3

Determine what the key terms refer to in the following study.
As part of a study designed to test the safety of automobiles, the National Transportation Safety Board collected and reviewed data about the effects of an automobile crash on test dummies. Here is the criterion they used:

| Speed at which Cars Crashed | Location of "drive" (i.e. dummies) |
| :--- | :--- |
| 35 miles/hour | Front Seat |

Table 1.1

Cars with dummies in the front seats were crashed into a wall at a speed of 35 miles per hour. We want to know the proportion of dummies in the driver's seat that would have had head injuries, if they had been actual drivers. We start with a simple random sample of 75 cars.

## Solution 1.3

The population is all cars containing dummies in the front seat.
The sample is the 75 cars, selected by a simple random sample.
The parameter is the proportion of driver dummies (if they had been real people) who would have suffered head injuries in the population.
The statistic is proportion of driver dummies (if they had been real people) who would have suffered head injuries in the sample.

The variable $X=$ the number of driver dummies (if they had been real people) who would have suffered head injuries.

The data are either: yes, had head injury, or no, did not.

## Example 1.4

Determine what the key terms refer to in the following study.
An insurance company would like to determine the proportion of all medical doctors who have been involved in one or more malpractice lawsuits. The company selects 500 doctors at random from a professional directory and determines the number in the sample who have been involved in a malpractice lawsuit.

## Solution 1.4

The population is all medical doctors listed in the professional directory.
The parameter is the proportion of medical doctors who have been involved in one or more malpractice suits in the population.
The sample is the 500 doctors selected at random from the professional directory.
The statistic is the proportion of medical doctors who have been involved in one or more malpractice suits in the sample.
The variable $X=$ the number of medical doctors who have been involved in one or more malpractice suits.
The data are either: yes, was involved in one or more malpractice lawsuits, or no, was not.

## 参Collaborative Exercise

Do the following exercise collaboratively with up to four people per group. Find a population, a sample, the parameter, the statistic, a variable, and data for the following study: You want to determine the average (mean) number of glasses of milk college students drink per day. Suppose yesterday, in your English class, you asked five students how many glasses of milk they drank the day before. The answers were $1,0,1,3$, and 4 glasses of milk.

## 1.2 | Data, Sampling, and Variation in Data and Sampling

Data may come from a population or from a sample. Small letters like $x$ or $y$ generally are used to represent data values. Most data can be put into the following categories:

- Qualitative
- Quantitative

Qualitative data are the result of categorizing or describing attributes of a population. Hair color, blood type, ethnic group, the car a person drives, and the street a person lives on are examples of qualitative data. Qualitative data are generally described by words or letters. For instance, hair color might be black, dark brown, light brown, blonde, gray, or red. Blood type might be $\mathrm{AB}+$, O -, or $\mathrm{B}+$. Researchers often prefer to use quantitative data over qualitative data because it lends itself more easily to mathematical analysis. For example, it does not make sense to find an average hair color or blood type.
Quantitative data are always numbers. Quantitative data are the result of counting or measuring attributes of a population. Amount of money, pulse rate, weight, number of people living in your town, and number of students who take statistics are examples of quantitative data. Quantitative data may be either discrete or continuous.

All data that are the result of counting are called quantitative discrete data. These data take on only certain numerical values. If you count the number of phone calls you receive for each day of the week, you might get values such as zero, one, two, or three.

All data that are the result of measuring are quantitative continuous data assuming that we can measure accurately. Measuring angles in radians might result in such numbers as $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3 \pi}{4}$, and so on. If you and your friends carry backpacks with books in them to school, the numbers of books in the backpacks are discrete data and the weights of the backpacks are continuous data.

## Example 1.5 Data Sample of Quantitative Discrete Data

The data are the number of books students carry in their backpacks. You sample five students. Two students carry three books, one student carries four books, one student carries two books, and one student carries one book. The numbers of books (three, four, two, and one) are the quantitative discrete data.

## Try It

1.5 The data are the number of machines in a gym. You sample five gyms. One gym has 12 machines, one gym has 15 machines, one gym has ten machines, one gym has 22 machines, and the other gym has 20 machines. What type of data is this?

## Example 1.6 Data Sample of Quantitative Continuous Data

The data are the weights of backpacks with books in them. You sample the same five students. The weights (in pounds) of their backpacks are $6.2,7,6.8,9.1,4.3$. Notice that backpacks carrying three books can have different weights. Weights are quantitative continuous data because weights are measured.

## Try It

1.6 The data are the areas of lawns in square feet. You sample five houses. The areas of the lawns are 144 sq. feet, 160 sq. feet, 190 sq. feet, 180 sq. feet, and 210 sq. feet. What type of data is this?

## Example 1.7

You go to the supermarket and purchase three cans of soup (19 ounces) tomato bisque, 14.1 ounces lentil, and 19 ounces Italian wedding), two packages of nuts (walnuts and peanuts), four different kinds of vegetable (broccoli, cauliflower, spinach, and carrots), and two desserts (16 ounces Cherry Garcia ice cream and two pounds (32 ounces chocolate chip cookies).
Name data sets that are quantitative discrete, quantitative continuous, and qualitative.

## Solution 1.7

One Possible Solution:

- The three cans of soup, two packages of nuts, four kinds of vegetables and two desserts are quantitative discrete data because you count them.
- The weights of the soups (19 ounces, 14.1 ounces, 19 ounces) are quantitative continuous data because you measure weights as precisely as possible.
- Types of soups, nuts, vegetables and desserts are qualitative data because they are categorical.

Try to identify additional data sets in this example.

## Example 1.8

The data are the colors of backpacks. Again, you sample the same five students. One student has a red backpack, two students have black backpacks, one student has a green backpack, and one student has a gray backpack. The colors red, black, black, green, and gray are qualitative data.

## Try It $\Sigma$

1.8 The data are the colors of houses. You sample five houses. The colors of the houses are white, yellow, white, red, and white. What type of data is this?

## NOTE

You may collect data as numbers and report it categorically. For example, the quiz scores for each student are recorded throughout the term. At the end of the term, the quiz scores are reported as A, B, C, D, or F.

## Example 1.9

Work collaboratively to determine the correct data type (quantitative or qualitative). Indicate whether quantitative data are continuous or discrete. Hint: Data that are discrete often start with the words "the number of."
a. the number of pairs of shoes you own
b. the type of car you drive
c. where you go on vacation
d. the distance it is from your home to the nearest grocery store
e. the number of classes you take per school year.
f. the tuition for your classes
g. the type of calculator you use
h. movie ratings
i. political party preferences
j. weights of sumo wrestlers
k. amount of money (in dollars) won playing poker
l. number of correct answers on a quiz
m. peoples' attitudes toward the government
n. IQ scores (This may cause some discussion.)

## Solution 1.9

Items $\mathrm{a}, \mathrm{e}, \mathrm{f}, \mathrm{k}$, and l are quantitative discrete; items $\mathrm{d}, \mathrm{j}$, and n are quantitative continuous; items $\mathrm{b}, \mathrm{c}, \mathrm{g}, \mathrm{h}, \mathrm{i}$, and m are qualitative.

## Try It $\Sigma$

1.9 Determine the correct data type (quantitative or qualitative) for the number of cars in a parking lot. Indicate whether quantitative data are continuous or discrete.

## Example 1.10

A statistics professor collects information about the classification of her students as freshmen, sophomores, juniors, or seniors. The data she collects are summarized in the pie chart Figure 1.2. What type of data does this graph show?

## Classification of Statistics Students



Figure 1.3

## Solution 1.10

This pie chart shows the students in each year, which is qualitative data.

## Try It $\Sigma$

1.10 The registrar at State University keeps records of the number of credit hours students complete each semester. The data he collects are summarized in the histogram. The class boundaries are 10 to less than 13,13 to less than 16 , 16 to less than 19, 19 to less than 22, and 22 to less than 25.

## Number of Credit Hours Completed per Students



Figure 1.4

What type of data does this graph show?

## Qualitative Data Discussion

Below are tables comparing the number of part-time and full-time students at De Anza College and Foothill College enrolled for the spring 2010 quarter. The tables display counts (frequencies) and percentages or proportions (relative frequencies). The percent columns make comparing the same categories in the colleges easier. Displaying percentages along with the numbers is often helpful, but it is particularly important when comparing sets of data that do not have the same totals, such as the total enrollments for both colleges in this example. Notice how much larger the percentage for part-time students at Foothill College is compared to De Anza College.

| De Anza College |  |  | Foothill College |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Number | Percent |  |  | Number | Percent |
| Full-time | 9,200 | $40.9 \%$ |  | Full-time | 4,059 | $28.6 \%$ |
| Part-time | 13,296 | $59.1 \%$ |  | Part-time | 10,124 | $71.4 \%$ |
| Total | 22,496 | $100 \%$ |  | Total | 14,183 | $100 \%$ |

Table 1.2 Fall Term 2007 (Census day)

Tables are a good way of organizing and displaying data. But graphs can be even more helpful in understanding the data. There are no strict rules concerning which graphs to use. Two graphs that are used to display qualitative data are pie charts and bar graphs.
In a pie chart, categories of data are represented by wedges in a circle and are proportional in size to the percent of individuals in each category.

In a bar graph, the length of the bar for each category is proportional to the number or percent of individuals in each category. Bars may be vertical or horizontal.
A Pareto chart consists of bars that are sorted into order by category size (largest to smallest).
Look at Figure 1.5 and Figure 1.6 and determine which graph (pie or bar) you think displays the comparisons better.
It is a good idea to look at a variety of graphs to see which is the most helpful in displaying the data. We might make different choices of what we think is the "best" graph depending on the data and the context. Our choice also depends on what we are using the data for.


Figure 1.5

Student Status


Figure 1.6

## Percentages That Add to More (or Less) Than 100\%

Sometimes percentages add up to be more than $100 \%$ (or less than $100 \%$ ). In the graph, the percentages add to more than $100 \%$ because students can be in more than one category. A bar graph is appropriate to compare the relative size of the categories. A pie chart cannot be used. It also could not be used if the percentages added to less than $100 \%$.

| Characteristic/Category | Percent |
| :--- | :--- |
| Full-Time Students | $40.9 \%$ |
| Students who intend to transfer to a 4-year educational institution | $48.6 \%$ |
| Students under age 25 | $61.0 \%$ |
| TOTAL | $150.5 \%$ |

Table 1.3 De Anza College Spring 2010


Figure 1.7

## Omitting Categories/Missing Data

The table displays Ethnicity of Students but is missing the "Other/Unknown" category. This category contains people who did not feel they fit into any of the ethnicity categories or declined to respond. Notice that the frequencies do not add up to the total number of students. In this situation, create a bar graph and not a pie chart.

|  | Frequency | Percent |
| :--- | :--- | :--- |
| Asian | 8,794 | $36.1 \%$ |
| Black | 1,412 | $5.8 \%$ |
| Filipino | 1,298 | $5.3 \%$ |
| Hispanic | 4,180 | $17.1 \%$ |
| Native American | 146 | $0.6 \%$ |
| Pacific Islander | 236 | $1.0 \%$ |
| White | 5,978 | $24.5 \%$ |
| TOTAL | 22,044 out of 24,382 | $90.4 \%$ out of $100 \%$ |

Table 1.4 Ethnicity of Students at De Anza College Fall Term 2007 (Census Day)


Figure 1.8

The following graph is the same as the previous graph but the "Other/Unknown" percent (9.6\%) has been included. The "Other/Unknown" category is large compared to some of the other categories (Native American, $0.6 \%$, Pacific Islander $1.0 \%)$. This is important to know when we think about what the data are telling us.

This particular bar graph in Figure 1.9 can be difficult to understand visually. The graph in Figure 1.10 is a Pareto chart. The Pareto chart has the bars sorted from largest to smallest and is easier to read and interpret.

Ethnicity of Students


Figure 1.9 Bar Graph with Other/Unknown Category

Ethnicity of Students


Figure 1.10 Pareto Chart With Bars Sorted by Size

## Pie Charts: No Missing Data

The following pie charts have the "Other/Unknown" category included (since the percentages must add to 100\%). The chart in Figure 1.11b is organized by the size of each wedge, which makes it a more visually informative graph than the unsorted, alphabetical graph in Figure 1.11a.


Figure 1.11

## Sampling

Gathering information about an entire population often costs too much or is virtually impossible. Instead, we use a sample of the population. A sample should have the same characteristics as the population it is representing. Most statisticians use various methods of random sampling in an attempt to achieve this goal. This section will describe a few of the most common methods. There are several different methods of random sampling. In each form of random sampling, each member of a population initially has an equal chance of being selected for the sample. Each method has pros and cons. The easiest method to describe is called a simple random sample. Any group of $n$ individuals is equally likely to be chosen by any other group of $n$ individuals if the simple random sampling technique is used. In other words, each sample of the same size has an equal chance of being selected. For example, suppose Lisa wants to form a four-person study group (herself and three other people) from her pre-calculus class, which has 31 members not including Lisa. To choose a simple random sample of size three from the other members of her class, Lisa could put all 31 names in a hat, shake the hat, close her eyes, and pick out three names. A more technological way is for Lisa to first list the last names of the members of her class together with a two-digit number, as in Table 1.5:

| ID | Name | ID | Name | ID | Name |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 00 | Anselmo | 11 | King | 21 | Roquero |
| 01 | Bautista | 12 | Legeny | 22 | Roth |
| 02 | Bayani | 13 | Lundquist | 23 | Rowell |
| 03 | Cheng | 14 | Macierz | 24 | Salangsang |
| 04 | Cuarismo | 15 | Motogawa | 25 | Slade |
| 05 | Cuningham | 16 | Okimoto | 26 | Stratcher |
| 06 | Fontecha | 17 | Patel | 27 | Tallai |
| 07 | Hong | 18 | Price | 28 | Tran |
| 08 | Hoobler | 19 | Quizon | 29 | Wai |
| 09 | Jiao | 20 | Reyes | 30 | Wood |
| 10 | Khan |  |  |  |  |

Table 1.5 Class Roster

Lisa can use a table of random numbers (found in many statistics books and mathematical handbooks), a calculator, or a computer to generate random numbers. For this example, suppose Lisa chooses to generate random numbers from a calculator. The numbers generated are as follows:
0.94360; 0.99832; 0.14669; 0.51470; 0.40581; 0.73381; 0.04399

Lisa reads two-digit groups until she has chosen three class members (that is, she reads 0.94360 as the groups $94,43,36$, 60 ). Each random number may only contribute one class member. If she needed to, Lisa could have generated more random numbers.
The random numbers 0.94360 and 0.99832 do not contain appropriate two digit numbers. However the third random number, 0.14669 , contains 14 (the fourth random number also contains 14 ), the fifth random number contains 05 , and the seventh random number contains 04 . The two-digit number 14 corresponds to Macierz, 05 corresponds to Cuningham, and 04 corresponds to Cuarismo. Besides herself, Lisa’s group will consist of Marcierz, Cuningham, and Cuarismo.

Using the $\mathrm{r}-83,83+, 84,84+$ Calculater

To generate random numbers:

- Press MATH.
- Arrow over to PRB.
- Press 5:randInt(. Enter 0, 30).
- Press ENTER for the first random number.
- Press ENTER two more times for the other 2 random numbers. If there is a repeat press ENTER again.

Note: randInt( $0,30,3$ ) will generate 3 random numbers.


Figure 1.12

Besides simple random sampling, there are other forms of sampling that involve a chance process for getting the sample. Other well-known random sampling methods are the stratified sample, the cluster sample, and the systematic sample.

To choose a stratified sample, divide the population into groups called strata and then take a proportionate number from each stratum. For example, you could stratify (group) your college population by department and then choose a proportionate simple random sample from each stratum (each department) to get a stratified random sample. To choose a simple random sample from each department, number each member of the first department, number each member of the second department, and do the same for the remaining departments. Then use simple random sampling to choose proportionate numbers from the first department and do the same for each of the remaining departments. Those numbers picked from the first department, picked from the second department, and so on represent the members who make up the stratified sample.
To choose a cluster sample, divide the population into clusters (groups) and then randomly select some of the clusters. All the members from these clusters are in the cluster sample. For example, if you randomly sample four departments from your college population, the four departments make up the cluster sample. Divide your college faculty by department. The departments are the clusters. Number each department, and then choose four different numbers using simple random sampling. All members of the four departments with those numbers are the cluster sample.

To choose a systematic sample, randomly select a starting point and take every $n^{\text {th }}$ piece of data from a listing of the population. For example, suppose you have to do a phone survey. Your phone book contains 20,000 residence listings. You must choose 400 names for the sample. Number the population $1-20,000$ and then use a simple random sample to pick a number that represents the first name in the sample. Then choose every fiftieth name thereafter until you have a total of 400 names (you might have to go back to the beginning of your phone list). Systematic sampling is frequently chosen because it is a simple method.
A type of sampling that is non-random is convenience sampling. Convenience sampling involves using results that are readily available. For example, a computer software store conducts a marketing study by interviewing potential customers who happen to be in the store browsing through the available software. The results of convenience sampling may be very good in some cases and highly biased (favor certain outcomes) in others.

Sampling data should be done very carefully. Collecting data carelessly can have devastating results. Surveys mailed to households and then returned may be very biased (they may favor a certain group). It is better for the person conducting the survey to select the sample respondents.

True random sampling is done with replacement. That is, once a member is picked, that member goes back into the population and thus may be chosen more than once. However for practical reasons, in most populations, simple random sampling is done without replacement. Surveys are typically done without replacement. That is, a member of the population may be chosen only once. Most samples are taken from large populations and the sample tends to be small in comparison to the population. Since this is the case, sampling without replacement is approximately the same as sampling with replacement because the chance of picking the same individual more than once with replacement is very low.
In a college population of 10,000 people, suppose you want to pick a sample of 1,000 randomly for a survey. For any particular sample of $\mathbf{1 , 0 0 0}$, if you are sampling with replacement,

- the chance of picking the first person is 1,000 out of $10,000(0.1000)$;
- the chance of picking a different second person for this sample is 999 out of 10,000 (0.0999);
- the chance of picking the same person again is 1 out of 10,000 (very low).

If you are sampling without replacement,

- the chance of picking the first person for any particular sample is 1000 out of $10,000(0.1000)$;
- the chance of picking a different second person is 999 out of 9,999 (0.0999);
- you do not replace the first person before picking the next person.

Compare the fractions 999/10,000 and 999/9,999. For accuracy, carry the decimal answers to four decimal places. To four decimal places, these numbers are equivalent (0.0999).
Sampling without replacement instead of sampling with replacement becomes a mathematical issue only when the population is small. For example, if the population is 25 people, the sample is ten, and you are sampling with replacement for any particular sample, then the chance of picking the first person is ten out of 25 , and the chance of picking a different second person is nine out of 25 (you replace the first person).
If you sample without replacement, then the chance of picking the first person is ten out of 25 , and then the chance of picking the second person (who is different) is nine out of 24 (you do not replace the first person).

Compare the fractions $9 / 25$ and $9 / 24$. To four decimal places, $9 / 25=0.3600$ and $9 / 24=0.3750$. To four decimal places, these numbers are not equivalent.
When you analyze data, it is important to be aware of sampling errors and nonsampling errors. The actual process of sampling causes sampling errors. For example, the sample may not be large enough. Factors not related to the sampling process cause nonsampling errors. A defective counting device can cause a nonsampling error.

In reality, a sample will never be exactly representative of the population so there will always be some sampling error. As a rule, the larger the sample, the smaller the sampling error.

In statistics, a sampling bias is created when a sample is collected from a population and some members of the population are not as likely to be chosen as others (remember, each member of the population should have an equally likely chance of being chosen). When a sampling bias happens, there can be incorrect conclusions drawn about the population that is being studied.

## Example 1.11

A study is done to determine the average tuition that San Jose State undergraduate students pay per semester. Each student in the following samples is asked how much tuition he or she paid for the Fall semester. What is the type of sampling in each case?
a. A sample of 100 undergraduate San Jose State students is taken by organizing the students’ names by classification (freshman, sophomore, junior, or senior), and then selecting 25 students from each.
b. A random number generator is used to select a student from the alphabetical listing of all undergraduate students in the Fall semester. Starting with that student, every 50th student is chosen until 75 students are included in the sample.
c. A completely random method is used to select 75 students. Each undergraduate student in the fall semester has the same probability of being chosen at any stage of the sampling process.
d. The freshman, sophomore, junior, and senior years are numbered one, two, three, and four, respectively. A random number generator is used to pick two of those years. All students in those two years are in the sample.
e. An administrative assistant is asked to stand in front of the library one Wednesday and to ask the first 100 undergraduate students he encounters what they paid for tuition the Fall semester. Those 100 students are the sample.

## Solution 1.11

a. stratified; b. systematic; c. simple random; d. cluster; e. convenience

## Try It

1.11 You are going to use the random number generator to generate different types of samples from the data.

This table displays six sets of quiz scores (each quiz counts 10 points) for an elementary statistics class.

| \#1 | \#2 | \#3 | \#4 | \#5 | \#6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 7 | 10 | 9 | 8 | 3 |
| 10 | 5 | 9 | 8 | 7 | 6 |
| 9 | 10 | 8 | 6 | 7 | 9 |
| 9 | 10 | 10 | 9 | 8 | 9 |
| 7 | 8 | 9 | 5 | 7 | 4 |
| 9 | 9 | 9 | 10 | 8 | 7 |
| 7 | 7 | 10 | 9 | 8 | 8 |
| 8 | 8 | 9 | 10 | 8 | 8 |
| 9 | 7 | 8 | 7 | 7 | 8 |
| 8 | 8 | 10 | 9 | 8 | 7 |

Table 1.6

Instructions: Use the Random Number Generator to pick samples.

1. Create a stratified sample by column. Pick three quiz scores randomly from each column.

- Number each row one through ten.
- On your calculator, press Math and arrow over to PRB.
- For column 1, Press 5:randInt( and enter 1,10). Press ENTER. Record the number. Press ENTER 2 more times (even the repeats). Record these numbers. Record the three quiz scores in column one that correspond to these three numbers.
- Repeat for columns two through six.
- These 18 quiz scores are a stratified sample.

2. Create a cluster sample by picking two of the columns. Use the column numbers: one through six.

- Press MATH and arrow over to PRB.
- Press 5:randInt( and enter 1,6). Press ENTER. Record the number. Press ENTER and record that number.
- The two numbers are for two of the columns.
- The quiz scores ( 20 of them) in these 2 columns are the cluster sample.

3. Create a simple random sample of 15 quiz scores.

- Use the numbering one through 60.
- Press MATH. Arrow over to PRB. Press 5:randInt( and enter 1, 60).
- Press ENTER 15 times and record the numbers.
- Record the quiz scores that correspond to these numbers.
- These 15 quiz scores are the systematic sample.

4. Create a systematic sample of 12 quiz scores.

- Use the numbering one through 60.
- Press MATH. Arrow over to PRB. Press 5:randInt( and enter 1, 60).
- Press ENTER. Record the number and the first quiz score. From that number, count ten quiz scores and record that quiz score. Keep counting ten quiz scores and recording the quiz score until you have a sample of 12 quiz scores. You may wrap around (go back to the beginning).


## Example 1.12

Determine the type of sampling used (simple random, stratified, systematic, cluster, or convenience).
a. A soccer coach selects six players from a group of boys aged eight to ten, seven players from a group of boys aged 11 to 12, and three players from a group of boys aged 13 to 14 to form a recreational soccer team.
b. A pollster interviews all human resource personnel in five different high tech companies.
c. A high school educational researcher interviews 50 high school female teachers and 50 high school male teachers.
d. A medical researcher interviews every third cancer patient from a list of cancer patients at a local hospital.
e. A high school counselor uses a computer to generate 50 random numbers and then picks students whose names correspond to the numbers.
f. A student interviews classmates in his algebra class to determine how many pairs of jeans a student owns, on the average.

## Solution 1.12

a. stratified; b. cluster; c. stratified; d. systematic; e. simple random; f.convenience

## Try It

1.12 Determine the type of sampling used (simple random, stratified, systematic, cluster, or convenience).

A high school principal polls 50 freshmen, 50 sophomores, 50 juniors, and 50 seniors regarding policy changes for after school activities.

If we were to examine two samples representing the same population, even if we used random sampling methods for the samples, they would not be exactly the same. Just as there is variation in data, there is variation in samples. As you become accustomed to sampling, the variability will begin to seem natural.

## Example 1.13

Suppose ABC College has 10,000 part-time students (the population). We are interested in the average amount of money a part-time student spends on books in the fall term. Asking all 10,000 students is an almost impossible task.
Suppose we take two different samples.
First, we use convenience sampling and survey ten students from a first term organic chemistry class. Many of these students are taking first term calculus in addition to the organic chemistry class. The amount of money they spend on books is as follows:
\$128; \$87; \$173; \$116; \$130; \$204; \$147; \$189; \$93; \$153
The second sample is taken using a list of senior citizens who take P.E. classes and taking every fifth senior citizen on the list, for a total of ten senior citizens. They spend:
\$50; \$40; \$36; \$15; \$50; \$100; \$40; \$53; \$22; \$22
It is unlikely that any student is in both samples.
a. Do you think that either of these samples is representative of (or is characteristic of) the entire 10,000 part-time student population?

## Solution 1.13

a. No. The first sample probably consists of science-oriented students. Besides the chemistry course, some of them are also taking first-term calculus. Books for these classes tend to be expensive. Most of these students are, more than likely, paying more than the average part-time student for their books. The second sample is a group of senior citizens who are, more than likely, taking courses for health and interest. The amount of money they spend on books is probably much less than the average parttime student. Both samples are biased. Also, in both cases, not all students have a chance to be in either sample.
b. Since these samples are not representative of the entire population, is it wise to use the results to describe the entire population?

## Solution 1.13

b. No. For these samples, each member of the population did not have an equally likely chance of being chosen.

Now, suppose we take a third sample. We choose ten different part-time students from the disciplines of chemistry, math, English, psychology, sociology, history, nursing, physical education, art, and early childhood development. (We assume that these are the only disciplines in which part-time students at ABC College are enrolled and that an equal number of part-time students are enrolled in each of the disciplines.) Each student is chosen using simple random sampling. Using a calculator, random numbers are generated and a student from a particular discipline is selected if he or she has a corresponding number. The students spend the following amounts:
\$180; \$50; \$150; \$85; \$260; \$75; \$180; \$200; \$200; \$150
c. Is the sample biased?

## Solution 1.13

c. The sample is unbiased, but a larger sample would be recommended to increase the likelihood that the sample will be close to representative of the population. However, for a biased sampling technique, even a large sample runs the risk of not being representative of the population.
Students often ask if it is "good enough" to take a sample, instead of surveying the entire population. If the survey is done well, the answer is yes.

## Try It

1.13 A local radio station has a fan base of 20,000 listeners. The station wants to know if its audience would prefer more music or more talk shows. Asking all 20,000 listeners is an almost impossible task.

The station uses convenience sampling and surveys the first 200 people they meet at one of the station's music concert events. 24 people said they'd prefer more talk shows, and 176 people said they'd prefer more music.
Do you think that this sample is representative of (or is characteristic of) the entire 20,000 listener population?

## EnCollaborative Exercise

As a class, determine whether or not the following samples are representative. If they are not, discuss the reasons.

1. To find the average GPA of all students in a university, use all honor students at the university as the sample.
2. To find out the most popular cereal among young people under the age of ten, stand outside a large supermarket for three hours and speak to every twentieth child under age ten who enters the supermarket.
3. To find the average annual income of all adults in the United States, sample U.S. congressmen. Create a cluster sample by considering each state as a stratum (group). By using simple random sampling, select states to be part of the cluster. Then survey every U.S. congressman in the cluster.
4. To determine the proportion of people taking public transportation to work, survey 20 people in New York City. Conduct the survey by sitting in Central Park on a bench and interviewing every person who sits next to you.
5. To determine the average cost of a two-day stay in a hospital in Massachusetts, survey 100 hospitals across the state using simple random sampling.

## Variation in Data

Variation is present in any set of data. For example, 16-ounce cans of beverage may contain more or less than 16 ounces of liquid. In one study, eight 16 ounce cans were measured and produced the following amount (in ounces) of beverage:
15.8; 16.1; 15.2; 14.8; 15.8; 15.9; 16.0; 15.5

Measurements of the amount of beverage in a 16-ounce can may vary because different people make the measurements or because the exact amount, 16 ounces of liquid, was not put into the cans. Manufacturers regularly run tests to determine if the amount of beverage in a 16 -ounce can falls within the desired range.

Be aware that as you take data, your data may vary somewhat from the data someone else is taking for the same purpose. This is completely natural. However, if two or more of you are taking the same data and get very different results, it is time for you and the others to reevaluate your data-taking methods and your accuracy.

## Variation in Samples

It was mentioned previously that two or more samples from the same population, taken randomly, and having close to the same characteristics of the population will likely be different from each other. Suppose Doreen and Jung both decide
to study the average amount of time students at their college sleep each night. Doreen and Jung each take samples of 500 students. Doreen uses systematic sampling and Jung uses cluster sampling. Doreen's sample will be different from Jung's sample. Even if Doreen and Jung used the same sampling method, in all likelihood their samples would be different. Neither would be wrong, however.
Think about what contributes to making Doreen's and Jung's samples different.
If Doreen and Jung took larger samples (i.e. the number of data values is increased), their sample results (the average amount of time a student sleeps) might be closer to the actual population average. But still, their samples would be, in all likelihood, different from each other. This variability in samples cannot be stressed enough.

## Size of a Sample

The size of a sample (often called the number of observations) is important. The examples you have seen in this book so far have been small. Samples of only a few hundred observations, or even smaller, are sufficient for many purposes. In polling, samples that are from 1,200 to 1,500 observations are considered large enough and good enough if the survey is random and is well done. You will learn why when you study confidence intervals.

Be aware that many large samples are biased. For example, call-in surveys are invariably biased, because people choose to respond or not.

## =, Cl Collaborative Exercise

Divide into groups of two, three, or four. Your instructor will give each group one six-sided die. Try this experiment twice. Roll one fair die (six-sided) 20 times. Record the number of ones, twos, threes, fours, fives, and sixes you get in Table 1.7 and Table 1.8 ("frequency" is the number of times a particular face of the die occurs):

| Face on Die | Frequency |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

Table 1.7 First Experiment (20 rolls)

| Face on Die | Frequency |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

Table 1.8 Second Experiment (20 rolls)

Did the two experiments have the same results? Probably not. If you did the experiment a third time, do you expect the results to be identical to the first or second experiment? Why or why not?

Which experiment had the correct results? They both did. The job of the statistician is to see through the variability and draw appropriate conclusions.

## Critical Evaluation

We need to evaluate the statistical studies we read about critically and analyze them before accepting the results of the studies. Common problems to be aware of include

- Problems with samples: A sample must be representative of the population. A sample that is not representative of the population is biased. Biased samples that are not representative of the population give results that are inaccurate and not valid.
- Self-selected samples: Responses only by people who choose to respond, such as call-in surveys, are often unreliable.
- Sample size issues: Samples that are too small may be unreliable. Larger samples are better, if possible. In some situations, having small samples is unavoidable and can still be used to draw conclusions. Examples: crash testing cars or medical testing for rare conditions
- Undue influence: collecting data or asking questions in a way that influences the response
- Non-response or refusal of subject to participate: The collected responses may no longer be representative of the population. Often, people with strong positive or negative opinions may answer surveys, which can affect the results.
- Causality: A relationship between two variables does not mean that one causes the other to occur. They may be related (correlated) because of their relationship through a different variable.
- Self-funded or self-interest studies: A study performed by a person or organization in order to support their claim. Is the study impartial? Read the study carefully to evaluate the work. Do not automatically assume that the study is good, but do not automatically assume the study is bad either. Evaluate it on its merits and the work done.
- Misleading use of data: improperly displayed graphs, incomplete data, or lack of context
- Confounding: When the effects of multiple factors on a response cannot be separated. Confounding makes it difficult or impossible to draw valid conclusions about the effect of each factor.


## 1.3 | Frequency, Frequency Tables, and Levels of Measurement

Once you have a set of data, you will need to organize it so that you can analyze how frequently each datum occurs in the set. However, when calculating the frequency, you may need to round your answers so that they are as precise as possible.

## Answers and Rounding Off

A simple way to round off answers is to carry your final answer one more decimal place than was present in the original data. Round off only the final answer. Do not round off any intermediate results, if possible. If it becomes necessary to round off intermediate results, carry them to at least twice as many decimal places as the final answer. For example, the average of the three quiz scores four, six, and nine is 6.3, rounded off to the nearest tenth, because the data are whole numbers. Most answers will be rounded off in this manner.

It is not necessary to reduce most fractions in this course. Especially in Probability Topics, the chapter on probability, it is more helpful to leave an answer as an unreduced fraction.

## Levels of Measurement

The way a set of data is measured is called its level of measurement. Correct statistical procedures depend on a researcher being familiar with levels of measurement. Not every statistical operation can be used with every set of data. Data can be classified into four levels of measurement. They are (from lowest to highest level):

- Nominal scale level
- Ordinal scale level
- Interval scale level
- Ratio scale level

Data that is measured using a nominal scale is qualitative. Categories, colors, names, labels and favorite foods along with yes or no responses are examples of nominal level data. Nominal scale data are not ordered. For example, trying to classify people according to their favorite food does not make any sense. Putting pizza first and sushi second is not meaningful.
Smartphone companies are another example of nominal scale data. Some examples are Sony, Motorola, Nokia, Samsung and Apple. This is just a list and there is no agreed upon order. Some people may favor Apple but that is a matter of opinion. Nominal scale data cannot be used in calculations.

Data that is measured using an ordinal scale is similar to nominal scale data but there is a big difference. The ordinal scale data can be ordered. An example of ordinal scale data is a list of the top five national parks in the United States. The top five national parks in the United States can be ranked from one to five but we cannot measure differences between the data.
Another example of using the ordinal scale is a cruise survey where the responses to questions about the cruise are "excellent," "good," "satisfactory," and "unsatisfactory." These responses are ordered from the most desired response to the least desired. But the differences between two pieces of data cannot be measured. Like the nominal scale data, ordinal scale data cannot be used in calculations.

Data that is measured using the interval scale is similar to ordinal level data because it has a definite ordering but there is a difference between data. The differences between interval scale data can be measured though the data does not have a starting point.
Temperature scales like Celsius (C) and Fahrenheit (F) are measured by using the interval scale. In both temperature measurements, $40^{\circ}$ is equal to $100^{\circ}$ minus $60^{\circ}$. Differences make sense. But 0 degrees does not because, in both scales, 0 is not the absolute lowest temperature. Temperatures like $-10^{\circ} \mathrm{F}$ and $-15^{\circ} \mathrm{C}$ exist and are colder than 0 .

Interval level data can be used in calculations, but one type of comparison cannot be done. $80^{\circ} \mathrm{C}$ is not four times as hot as $20^{\circ} \mathrm{C}$ (nor is $80^{\circ} \mathrm{F}$ four times as hot as $20^{\circ} \mathrm{F}$ ). There is no meaning to the ratio of 80 to 20 (or four to one).
Data that is measured using the ratio scale takes care of the ratio problem and gives you the most information. Ratio scale data is like interval scale data, but it has a 0 point and ratios can be calculated. For example, four multiple choice statistics final exam scores are 80, 68, 20 and 92 (out of a possible 100 points). The exams are machine-graded.
The data can be put in order from lowest to highest: 20, 68, 80, 92.
The differences between the data have meaning. The score 92 is more than the score 68 by 24 points. Ratios can be calculated. The smallest score is 0 . So 80 is four times 20 . The score of 80 is four times better than the score of 20 .

## Frequency

Twenty students were asked how many hours they worked per day. Their responses, in hours, are as follows: 5; 6; 3; 3; 2; 4; 7; 5; 2; 3; 5; 6; 5; 4; 4; 3; 5; 2; 5; 3.

Table 1.9 lists the different data values in ascending order and their frequencies.

| DATA VALUE | FREQUENCY |
| :--- | :--- |
| 2 | 3 |
| 3 | 5 |
| 4 | 3 |
| 5 | 6 |
| 6 | 2 |
| 7 | 1 |

Table 1.9 Frequency Table of Student Work Hours

A frequency is the number of times a value of the data occurs. According to Table 1.9, there are three students who work two hours, five students who work three hours, and so on. The sum of the values in the frequency column, 20, represents the total number of students included in the sample.

A relative frequency is the ratio (fraction or proportion) of the number of times a value of the data occurs in the set of all outcomes to the total number of outcomes. To find the relative frequencies, divide each frequency by the total number of students in the sample-in this case, 20. Relative frequencies can be written as fractions, percents, or decimals.

| DATA VALUE | FREQUENCY | RELATIVE FREQUENCY |
| :--- | :--- | :--- |
| 2 | 3 | $\frac{3}{20}$ or 0.15 |
| 3 | 5 | $\frac{5}{20}$ or 0.25 |
| 4 | 3 | $\frac{3}{20}$ or 0.15 |
| 5 | 2 | $\frac{6}{20}$ or 0.30 |
| 6 | 1 | $\frac{2}{20}$ or 0.10 |
| 7 | 2 | $\frac{1}{20}$ or 0.05 |

Table 1.10 Frequency Table of Student Work Hours with Relative Frequencies

The sum of the values in the relative frequency column of Table 1.10 is $\frac{20}{20}$, or 1 .
Cumulative relative frequency is the accumulation of the previous relative frequencies. To find the cumulative relative frequencies, add all the previous relative frequencies to the relative frequency for the current row, as shown in Table 1.11.

| DATA VALUE | FREQUENCY | RELATIVE <br> FREQUENCY | CUMULATIVE RELATIVE <br> FREQUENCY |
| :--- | :--- | :--- | :--- |
| 2 | 3 | $\frac{3}{20}$ or 0.15 | 0.15 |
| 3 | 5 | $\frac{5}{20}$ or 0.25 | $0.15+0.25=0.40$ |
| 4 | 3 | $\frac{3}{20}$ or 0.15 | $0.40+0.15=0.55$ |
| 5 | 6 | $\frac{6}{20}$ or 0.30 | $0.55+0.30=0.85$ |
| 6 | 1 | $\frac{2}{20}$ or 0.10 | $0.85+0.10=0.95$ |
| 7 | 1 | $\frac{1}{20}$ or 0.05 | $0.95+0.05=1.00$ |

Table 1.11 Frequency Table of Student Work Hours with Relative and Cumulative Relative Frequencies

The last entry of the cumulative relative frequency column is one, indicating that one hundred percent of the data has been accumulated.

## NOTE

Because of rounding, the relative frequency column may not always sum to one, and the last entry in the cumulative relative frequency column may not be one. However, they each should be close to one.

Table 1.12 represents the heights, in inches, of a sample of 100 male semiprofessional soccer players.

| HEIGHTS <br> (INCHES) | FREQUENCY | RELATIVE <br> FREQUENCY | CUMULATIVE <br> RELATIVE <br> FREQUENCY |
| :--- | :--- | :--- | :--- |
| $59.95-61.95$ | 5 | $\frac{5}{100}=0.05$ | 0.05 |
| $61.95-63.95$ | 3 | $\frac{3}{100}=0.03$ | $0.05+0.03=0.08$ |
| $63.95-65.95$ | 15 | $\frac{15}{100}=0.15$ | $0.08+0.15=0.23$ |
| $65.95-67.95$ | 40 | $\frac{40}{100}=0.40$ | $0.23+0.40=0.63$ |
| $67.95-69.95$ | 17 | $\frac{17}{100}=0.17$ | $0.63+0.17=0.80$ |
| $69.95-71.95$ | 12 | $\frac{12}{100}=0.12$ | $0.80+0.12=0.92$ |
| $71.95-73.95$ | 7 | $\frac{7}{100}=0.07$ | $0.92+0.07=0.99$ |
| $73.95-75.95$ | 1 | $\frac{1}{100}=0.01$ | $0.99+0.01=1.00$ |
|  | Total $=100$ | Total $=1.00$ |  |

Table 1.12 Frequency Table of Soccer Player Height

The data in this table have been grouped into the following intervals:

- 59.95 to 61.95 inches
- 61.95 to 63.95 inches
- 63.95 to 65.95 inches
- 65.95 to 67.95 inches
- 67.95 to 69.95 inches
- 69.95 to 71.95 inches
- 71.95 to 73.95 inches
- 73.95 to 75.95 inches


## NOTE

This example is used again in Descriptive Statistics, where the method used to compute the intervals will be explained.

In this sample, there are five players whose heights fall within the interval 59.95-61.95 inches, three players whose heights fall within the interval 61.95-63.95 inches, $\mathbf{1 5}$ players whose heights fall within the interval 63.95-65.95 inches, $\mathbf{4 0}$ players whose heights fall within the interval 65.95-67.95 inches, 17 players whose heights fall within the interval 67.95-69.95 inches, 12 players whose heights fall within the interval 69.95-71.95, seven players whose heights fall within the interval 71.95-73.95, and one player whose heights fall within the interval 73.95-75.95. All heights fall between the endpoints of an interval and not at the endpoints.

## Example 1.14

From Table 1.12, find the percentage of heights that are less than 65.95 inches.

## Solution 1.14

If you look at the first, second, and third rows, the heights are all less than 65.95 inches. There are $5+3+15=23$ players whose heights are less than 65.95 inches. The percentage of heights less than 65.95 inches is then $\frac{23}{100}$ or $23 \%$. This percentage is the cumulative relative frequency entry in the third row.

## Try It $\Sigma$

1.14 Table 1.13 shows the amount, in inches, of annual rainfall in a sample of towns.

| Rainfall (Inches) | Frequency | Relative Frequency | Cumulative Relative Frequency |
| :--- | :--- | :--- | :--- |
| $2.95-4.97$ | 6 | $\frac{6}{50}=0.12$ | 0.12 |
| $4.97-6.99$ | 7 | $\frac{7}{50}=0.14$ | $0.12+0.14=0.26$ |
| $6.99-9.01$ | 15 | $\frac{15}{50}=0.30$ | $0.26+0.30=0.56$ |
| $9.01-11.03$ | 8 | $\frac{8}{50}=0.16$ | $0.56+0.16=0.72$ |
| $11.03-13.05$ | 9 | $\frac{9}{50}=0.18$ | $0.72+0.18=0.90$ |
| $13.05-15.07$ | 5 | $\frac{5}{50}=0.10$ | $0.90+0.10=1.00$ |
|  | Total $=50$ | Total $=1.00$ |  |

Table 1.13

From Table 1.13, find the percentage of rainfall that is less than 9.01 inches.

## Example 1.15

From Table 1.12, find the percentage of heights that fall between 61.95 and 65.95 inches.

## Solution 1.15

Add the relative frequencies in the second and third rows: $0.03+0.15=0.18$ or $18 \%$.

## Try It $\Sigma$

1.15 From Table 1.13, find the percentage of rainfall that is between 6.99 and 13.05 inches.

## Example 1.16

Use the heights of the 100 male semiprofessional soccer players in Table 1.12. Fill in the blanks and check your answers.
a. The percentage of heights that are from 67.95 to 71.95 inches is: $\qquad$ -
b. The percentage of heights that are from 67.95 to 73.95 inches is: $\qquad$ -
c. The percentage of heights that are more than 65.95 inches is: $\qquad$ -.
d. The number of players in the sample who are between 61.95 and 71.95 inches tall is: $\qquad$ -
e. What kind of data are the heights?
f. Describe how you could gather this data (the heights) so that the data are characteristic of all male semiprofessional soccer players.
Remember, you count frequencies. To find the relative frequency, divide the frequency by the total number of data values. To find the cumulative relative frequency, add all of the previous relative frequencies to the relative frequency for the current row.

## Solution 1.16

a. $29 \%$
b. $36 \%$
c. $77 \%$
d. 87
e. quantitative continuous
f. get rosters from each team and choose a simple random sample from each

## Try It

1.16 From Table 1.13, find the number of towns that have rainfall between 2.95 and 9.01 inches.

## , N Collaborative Exercise

In your class, have someone conduct a survey of the number of siblings (brothers and sisters) each student has. Create a frequency table. Add to it a relative frequency column and a cumulative relative frequency column. Answer the following questions:

1. What percentage of the students in your class have no siblings?
2. What percentage of the students have from one to three siblings?
3. What percentage of the students have fewer than three siblings?

## Example 1.17

Nineteen people were asked how many miles, to the nearest mile, they commute to work each day. The data are as follows: $2 ; 5 ; 7 ; 3 ; 2 ; 10 ; 18 ; 15 ; 20 ; 7 ; 10 ; 18 ; 5 ; 12 ; 13 ; 12 ; 4 ; 5 ; 10$. Table 1.14 was produced:

| DATA | FREQUENCY | RELATIVE <br> FREQUENCY | CUMULATIVE <br> RELATIVE <br> FREQUENCY |
| :--- | :--- | :--- | :--- |
| 3 | 3 | $\frac{3}{19}$ | 0.1579 |
| 4 | 1 | $\frac{1}{19}$ | 0.2105 |
| 5 | 3 | $\frac{3}{19}$ | 0.1579 |
| 7 | 2 | $\frac{2}{19}$ | 0.2632 |
| 10 | 3 | $\frac{4}{19}$ | 0.4737 |
| 12 | 2 | $\frac{1}{19}$ | 0.7895 |
| 13 | 1 | $\frac{1}{19}$ | 0.8421 |
| 15 | 1 | $\frac{1}{19}$ | 0.8948 |
| 18 | 1 | 1.0000 |  |
| 20 | 3 |  |  |

Table 1.14 Frequency of Commuting Distances
a. Is the table correct? If it is not correct, what is wrong?
b. True or False: Three percent of the people surveyed commute three miles. If the statement is not correct, what should it be? If the table is incorrect, make the corrections.
c. What fraction of the people surveyed commute five or seven miles?
d. What fraction of the people surveyed commute 12 miles or more? Less than 12 miles? Between five and 13 miles (not including five and 13 miles)?

## Solution 1.17

a. No. The frequency column sums to 18, not 19. Not all cumulative relative frequencies are correct.
b. False. The frequency for three miles should be one; for two miles (left out), two. The cumulative relative frequency column should read: $0.1052,0.1579,0.2105,0.3684,0.4737,0.6316,0.7368,0.7895,0.8421$, 0.9474, 1.0000.
c. $\frac{5}{19}$
d. $\frac{7}{19}, \frac{12}{19}, \frac{7}{19}$

## Try It

1.17 Table 1.13 represents the amount, in inches, of annual rainfall in a sample of towns. What fraction of towns surveyed get between 11.03 and 13.05 inches of rainfall each year?

## Example 1.18

Table 1.15 contains the total number of deaths worldwide as a result of earthquakes for the period from 2000 to 2012.

| Year | Total Number of Deaths |
| :--- | :--- |
| 2000 | 231 |
| 2001 | 21,357 |
| 2002 | 11,685 |
| 2003 | 33,819 |
| 2004 | 228,802 |
| 2005 | 88,003 |
| 2006 | 6,605 |
| 2007 | 712 |
| 2008 | 88,011 |
| 2009 | 1,790 |
| 2010 | 320,120 |
| 2011 | 21,953 |
| 2012 | 768 |
| Total | 823,356 |

Table 1.15

Answer the following questions.
a. What is the frequency of deaths measured from 2006 through 2009 ?
b. What percentage of deaths occurred after 2009?
c. What is the relative frequency of deaths that occurred in 2003 or earlier?
d. What is the percentage of deaths that occurred in 2004?
e. What kind of data are the numbers of deaths?
f. The Richter scale is used to quantify the energy produced by an earthquake. Examples of Richter scale numbers are $2.3,4.0,6.1$, and 7.0 . What kind of data are these numbers?

## Solution 1.18

a. 97,118 (11.8\%)
b. $41.6 \%$
c. $67,092 / 823,356$ or 0.081 or $8.1 \%$
d. $27.8 \%$
e. Quantitative discrete
f. Quantitative continuous

## Try It $\Sigma$

1.18 Table 1.16 contains the total number of fatal motor vehicle traffic crashes in the United States for the period from 1994 to 2011.

| Year | Total Number of Crashes | Year | Total Number of Crashes |
| :--- | :--- | :--- | :--- |
| 1994 | 36,254 | 2004 | 38,444 |
| 1995 | 37,241 | 2005 | 39,252 |
| 1996 | 37,494 | 2006 | 38,648 |
| 1997 | 37,324 | 2007 | 37,435 |
| 1998 | 37,107 | 2008 | 34,172 |
| 1999 | 37,140 | 2009 | 30,862 |
| 2000 | 37,526 | 2010 | 30,296 |
| 2001 | 37,862 | 2011 | 29,757 |
| 2002 | 38,491 | Total | 653,782 |
| 2003 | 38,477 |  |  |

Table 1.16

Answer the following questions.
a. What is the frequency of deaths measured from 2000 through 2004?
b. What percentage of deaths occurred after 2006?
c. What is the relative frequency of deaths that occurred in 2000 or before?
d. What is the percentage of deaths that occurred in 2011?
e. What is the cumulative relative frequency for 2006? Explain what this number tells you about the data.

