DECIMALS

Introduction

Gasoline prices change all the time. They might go down for a period of time, but then they usually rise again. One thing that stays the same is that the price is not usually a whole number. Instead, it is shown using a decimal point to describe the cost in dollars and cents. We use decimal numbers all the time, especially when dealing with money. In this chapter, we will explore decimal numbers and how to perform operations using them.

5.1 Decimals

Learning Objectives

By the end of this section, you will be able to:

- Name decimals
- Write decimals
- Convert decimals to fractions or mixed numbers
- Locate decimals on the number line
- Order decimals
- Round decimals

Name Decimals

You probably already know quite a bit about decimals based on your experience with money. Suppose you buy a sandwich and a bottle of water for lunch. If the sandwich costs $3.45, the bottle of water costs $1.25, and the total sales tax is $0.33, what is the total cost of your lunch?

\$3.45	Sandwich
\$1.25	Water
+ \$0.33	Tax
\$5.03	Total

The total is \$5.03. Suppose you pay with a \$5 bill and 3 pennies. Should you wait for change? No, \$5 and 3 pennies is the same as \$5.03.

Because 100 pennies = \$1, each penny is worth $\frac{1}{100}$ of a dollar. We write the value of one penny as \$0.01, since $0.01 = \frac{1}{100}$.

Writing a number with a decimal is known as decimal notation. It is a way of showing parts of a whole when the whole is a power of 10. In other words, decimals are another way of writing fractions whose denominators are powers of 10. Just as the counting numbers are based on powers of 10, decimals are based on powers of 10. Table 5.1 shows the counting numbers.

Counting number	Name
1	One
$10 = 10$	Ten
$10 \cdot 10 = 100$	One hundred
$10 \cdot 10 \cdot 10 = 1000$	One thousand
$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	Ten thousand

Table 5.1

How are decimals related to fractions? Table 5.2 shows the relation.

Counting number	Name
1	One
$10 = 10$	Ten
$10 \cdot 10 = 100$	One hundred
$10 \cdot 10 \cdot 10 = 1000$	One thousand
$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	Ten thousand

Table 5.2

When we name a whole number, the name corresponds to the place value based on the powers of ten. In Whole Numbers, we learned to read 10,000 as *ten thousand*. Likewise, the names of the decimal places correspond to their fraction values. Notice how the place value names in Figure 5.2 relate to the names of the fractions from Table 5.2.

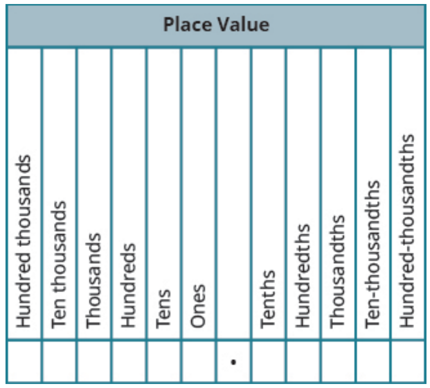


Figure 5.2 This chart illustrates place values to the left and right of the decimal point.

Notice two important facts shown in **Figure 5.2**.

- The "th" at the end of the name means the number is a fraction. "One thousand" is a number larger than one, but "one thousandth" is a number smaller than one.
- The tenths place is the first place to the right of the decimal, but the tens place is two places to the left of the decimal.

Remember that $5.03 lunch? We read $5.03 as *five dollars and three cents*. Naming decimals (those that don't represent money) is done in a similar way. We read the number 5.03 as *five and three hundredths.*

We sometimes need to translate a number written in decimal notation into words. As shown in **Figure 5.3**, we write the amount on a check in both words and numbers.

Figure 5.3 When we write a check, we write the amount as a decimal number as well as in words. The bank looks at the check to make sure both numbers match. This helps prevent errors.

Let's try naming a decimal, such as 15.68.	
We start by naming the number to the left of the decimal.	fifteen______
We use the word "and" to indicate the decimal point.	fifteen and_____
Then we name the number to the right of the decimal point as if it were a whole number.	fifteen and sixty-eight_____
Last, name the decimal place of the last digit.	fifteen and sixty-eight hundredths

The number 15.68 is read *fifteen and sixty-eight hundredths.*

? **HOW TO:** NAME A DECIMAL NUMBER

- Name the number to the left of the decimal point.

- Write "and" for the decimal point.
- Name the "number" part to the right of the decimal point as if it were a whole number.
- Name the decimal place of the last digit.

EXAMPLE 5.1

Name each decimal: ⓐ 4.3 ⓑ 2.45 ⓒ 0.009 ⓓ −15.571.

Solution

ⓐ

	4.3
Name the number to the left of the decimal point.	four______
Write "and" for the decimal point.	four and______
Name the number to the right of the decimal point as if it were a whole number.	four and three______
Name the decimal place of the last digit.	four and three tenths

ⓑ

	2.45
Name the number to the left of the decimal point.	two______
Write "and" for the decimal point.	two and______
Name the number to the right of the decimal point as if it were a whole number.	two and forty-five______
Name the decimal place of the last digit.	two and forty-five hundredths

ⓒ

	0.009
Name the number to the left of the decimal point.	Zero is the number to the left of the decimal; it is not included in the name.
Name the number to the right of the decimal point as if it were a whole number.	nine______
Name the decimal place of the last digit.	nine thousandths

ⓓ

	-15.571
Name the number to the left of the decimal point.	negative fifteen
Write "and" for the decimal point.	negative fifteen and______
Name the number to the right of the decimal point as if it were a whole number.	negative fifteen and five hundred seventy-one______
Name the decimal place of the last digit.	negative fifteen and five hundred seventy-one thousandths

TRY IT: 5.1 Name each decimal:

ⓐ 6.7 ⓑ 19.58 ⓒ 0.018 ⓓ −2.053

Solution

1. ⓐ six and seven tenths
2. ⓑ nineteen and fifty-eight hundredths
3. ⓒ eighteen thousandths
4. ⓓ negative two and fifty-three thousandths

TRY IT: 5.2 Name each decimal:

ⓐ 5.8 ⓑ 3.57 ⓒ 0.005 ⓓ −13.461

Solution

1. ⓐfive and eight tenths
2. ⓑthree and fifty-seven hundredths
3. ⓒ five thousandths
4. ⓓ negative thirteen and four hundred sixty-one thousandths

Write Decimals

Now we will translate the name of a decimal number into decimal notation. We will reverse the procedure we just used. Let's start by writing the number six and seventeen hundredths:

	six and seventeen hundredths
The word *and* tells us to place a decimal point.	___.___
The word before *and* is the whole number; write it to the left of the decimal point.	6._____
The decimal part is seventeen hundredths. Mark two places to the right of the decimal point for hundredths.	6._ _
Write the numerals for seventeen in the places marked.	6.17

EXAMPLE 5.2

Write fourteen and thirty-seven hundredths as a decimal.

Solution

	fourteen and thirty-seven hundredths
Place a decimal point under the word 'and'.	______. _________
Translate the words before 'and' into the whole number and place it to the left of the decimal point.	14. _________
Mark two places to the right of the decimal point for "hundredths".	14.__ __
Translate the words after "and" and write the number to the right of the decimal point.	14.37
	Fourteen and thirty-seven hundredths is written 14.37.

TRY IT: 5.3 Write as a decimal: thirteen and sixty-eight hundredths.

Solution

13.68

TRY IT: 5.4 Write as a decimal: five and eight hundred ninety-four thousandths.

Solution

5.894

HOW TO: WRITE A DECIMAL NUMBER FROM ITS NAME

Step 1. Look for the word "and"—it locates the decimal point.
Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.

- Place a decimal point under the word "and." Translate the words before "and" into the whole number and place it to the left of the decimal point.
- If there is no "and," write a "0" with a decimal point to its right.

Step 3. Translate the words after "and" into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.

Step 4. Fill in zeros for place holders as needed.

If there is no whole number, we write a 0 to the left of the decimal point as a placeholder. So we would write "nine tenths" as 0.9.

EXAMPLE 5.3

Write twenty-four thousandths as a decimal.

Solution

	twenty-four thousandths
Look for the word "and".	There is no "and" so start with 0 0.
To the right of the decimal point, put three decimal places for thousandths.	0. _ _ _ tenths hundredths thousandths
Write the number 24 with the 4 in the thousandths place.	0. _ 2 4 tenths hundredths thousandths
Put zeros as placeholders in the remaining decimal places.	0.024
	So, twenty-four thousandths is written 0.024

TRY IT: 5.5 Write as a decimal: fifty-eight thousandths.

Solution

0.058

TRY IT: 5.6 Write as a decimal: sixty-seven thousandths.

Solution

0.067

Before we move on to our next objective, think about money again. We know that $1 is the same as $1.00. The way we write $1 (or $1.00) depends on the context. In the same way, integers can be written as decimals with as many zeros as needed to the right of the decimal.

$$\begin{array}{ll} 5 = 5.0 & -2 = -2.0 \\ 5 = 5.00 & -2 = -2.00 \\ 5 = 5.000 & -2 = -2.000 \end{array}$$

and so on…

Convert Decimals to Fractions or Mixed Numbers

We often need to rewrite decimals as fractions or mixed numbers. Let's go back to our lunch order to see how we can convert decimal numbers to fractions. We know that $5.03 means 5 dollars and 3 cents. Since there are 100 cents in one dollar, 3 cents means $\frac{3}{100}$ of a dollar, so $0.03 = \frac{3}{100}$.

We convert decimals to fractions by identifying the place value of the farthest right digit. In the decimal 0.03, the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$0.03 = \frac{3}{100}$$

For our \$5.03 lunch, we can write the decimal 5.03 as a mixed number.

$$5.03 = 5\frac{3}{100}$$

Notice that when the number to the left of the decimal is zero, we get a proper fraction. When the number to the left of the decimal is not zero, we get a mixed number.

? HOW TO: CONVERT A DECIMAL NUMBER TO A FRACTION OR MIXED NUMBER

1. Look at the number to the left of the decimal.
    - If it is zero, the decimal converts to a proper fraction.
    - If it is not zero, the decimal converts to a mixed number.
        - Write the whole number.
2. Determine the place value of the final digit.
3. Write the fraction.
    - numerator—the "numbers" to the right of the decimal point
    - denominator—the place value corresponding to the final digit
4. Simplify the fraction, if possible.

EXAMPLE 5.4

Write each of the following decimal numbers as a fraction or a mixed number:
ⓐ4.094.09 ⓑ3.73.7 ⓒ−0.286

Solution

ⓐ	
	4.09
There is a 4 to the left of the decimal point. Write "4" as the whole number part of the mixed number.	$4\frac{\square}{\square}$
Determine the place value of the final digit.	4. 0 tenths 9 hundredths
Write the fraction. Write 9 in the numerator as it is the number to the right of the decimal point.	$4\frac{9}{\square}$
Write 100 in the denominator as the place value of the final digit, 9, is hundredth.	$4\frac{9}{100}$
The fraction is in simplest form.	So, $4.09 = 4\frac{9}{100}$

Did you notice that the number of zeros in the denominator is the same as the number of **decimal** places?

ⓑ	
	3.7
There is a 3 to the left of the decimal point. Write "3" as the whole number part of the mixed number.	$3\frac{\square}{\square}$
Determine the place value of the final digit.	3. 7 tenths
Write the fraction. Write 7 in the numerator as it is the number to the right of the decimal point.	$3\frac{7}{\square}$
Write 10 in the denominator as the place value of the final digit, 7, is tenths.	$3\frac{7}{10}$
The fraction is in simplest form.	So, $3.7 = 3\frac{7}{10}$

ⓒ	
	−0.286
There is a 0 to the left of the decimal point. Write a negative sign before the fraction.	$-\frac{\square}{\square}$
Determine the place value of the final digit and write it in the denominator.	−0. 2 8 6 tenths hundredths thousandths
Write the fraction. Write 286 in the numerator as it is the number to the right of the decimal point.	$-\frac{286}{1000}$
Write 1,000 in the denominator as the place value of the final digit, 6, is thousandths.	
We remove a common factor of 2 to simplify the fraction.	$-\frac{143}{500}$

TRY IT: 5.7 Write as a fraction or mixed number. Simplify the answer if possible.

ⓐ 5.3 ⓑ 6.07 ⓒ −0.234

Solution

ⓐ $5\frac{3}{10}$, ⓑ $6\frac{7}{100}$, ⓒ $-\frac{234}{1000}$

TRY IT: 5.8 Write as a fraction or mixed number. Simplify the answer if possible.

ⓐ 8.7 ⓑ 1.03 ⓒ −0.024

Solution

ⓐ $8\frac{7}{10}$, ⓑ $1\frac{3}{100}$, ⓒ $-\frac{24}{1000}$

Locate Decimals on the Number Line

Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

EXAMPLE 5.5

Locate 0.4 on a number line.

Solution

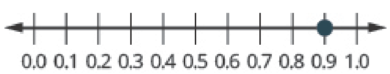
The decimal **0.4** is equivalent to **4**, so **0.4** is located between **0** and **1**. On a number line, divide the interval between **0** and **1** into **10** equal parts and place marks to separate the parts. Label the marks **0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0**. We write **0** as **0.0** and **1** as **1.0**, so that the numbers are consistently in tenths. Finally, mark **0.4** on the number line.

TRY IT: 5.09 Locate 0.6 on a number line.

Solution

TRY IT: 5.10 Locate 0.9 on a number line.

Solution

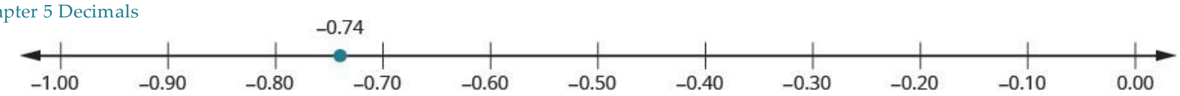


EXAMPLE 5.6

Locate −0.74 on a number line.

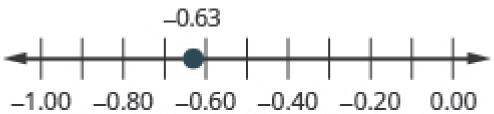
Solution

The decimal **−0.74** is equivalent to $-\frac{74}{100}$, so it is located between **0** and **−1**. On a number line, mark off and label the hundredths in the interval between **0** and **−1** (**−0.10, −0.20**, etc.) and mark **−0.74** between **−0.70** and **−0.80**, a little closer to **−0.70**.



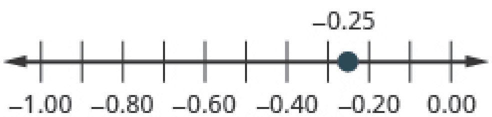
TRY IT: 5.11 Locate -0.63 on a number line.

Solution



TRY IT: 5.12 Locate −0.25 on a number line.

Solution



Order Decimals

Which is larger, 0.04 or 0.40?

If you think of this as money, you know that **$0.40** (forty cents) is greater than **$0.04** (four cents). So,

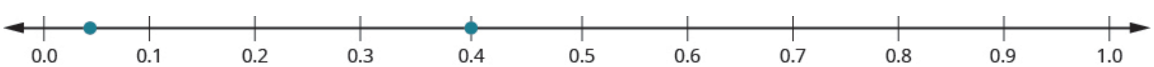
$$0.40 > 0.04.$$

In previous chapters, we used the number line to order numbers.

a < b "a is less than b" when a is to the left of b on the number line

a > b "a is greater than b" when a is to the right of b on the number line

Where are **0.04** and **0.40** located on the number line?



We see that **0.40** is to the right of **0.04**. So we know **0.40** > **0.04**.

How does 0.31 compare to 0.308? This doesn't translate into money to make the comparison easy. But if we convert and 0.308 to fractions, we can tell which is larger.

	0.31	0.308
Convert to fractions.	$\frac{31}{100}$	$\frac{308}{1000}$
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	$\frac{308}{1000}$
	$\frac{310}{1000}$	$\frac{308}{1000}$

Because 310 > 308, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, **0.31** > **0.308**.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives **0.310**. So **0.31** is equivalent to 0.310. Writing zeros at the end of a decimal does not change its value.

$$\frac{31}{100} = \frac{310}{1000} \text{ and } 0.31 = 0.310$$

If two decimals have the same value, they are said to be equivalent decimals.

$$0.31 = 0.310$$

We say 0.31 and 0.310 are equivalent decimals.

NOTE: EQUIVALENT DECIMALS
Two decimals are **equivalent decimals** if they convert to equivalent fractions.

Remember, writing zeros at the end of a decimal does not change its value.

? **HOW TO:** ORDER DECIMALS

Step 1. Check to see if both numbers have the same number of decimal places. If not, write zeros at the end of the one with fewer digits to make them match.

Step 2. Compare the numbers to the right of the decimal point as if they were whole numbers.

Step 3. Order the numbers using the appropriate inequality sign.

EXAMPLE 5.7

Order the following decimals using < or >:

ⓐ 0.64 _ 0.6

ⓑ 0.83 _0.803

Solution

ⓐ	
	0.64 __ 0.6
Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.6.	0.64 __ 0.60
Compare the numbers to the right of the decimal point as if they were whole numbers.	$64 > 60$
Order the numbers using the appropriate inequality sign.	$0.64 > 0.60$
	$0.64 > 0.6$
ⓑ	
	0.83 __ 0.803
Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.83.	0.830 __ 0.803
Compare the numbers to the right of the decimal point as if they were whole numbers.	$830 > 803$
Order the numbers using the appropriate inequality sign.	$0.830 > 0.803$
	$0.83 > 0.803$

TRY IT: 5.13 Order each of the following pairs of numbers, using < or >:

ⓐ 0.42 0.4 ⓑ 0.76 0.706

Solution

ⓐ >

ⓑ >

TRY IT: 5.14 Order each of the following pairs of numbers, using < or >:

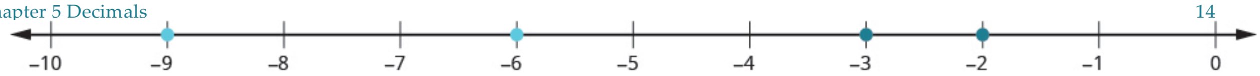
ⓐ 0.1 0.18 ⓑ 0.305 0.35

Solution

ⓐ <

ⓑ <

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because **−2** lies to the right of **−3** on the number line, we know that **−2** > **−3**. Similarly, smaller numbers lie to the left on the number line. For example, because **−9** lies to the left of **−6** on the number line, we know that **−9** < **−6**.



If we zoomed in on the interval between **0** and **−1**, we would see in the same way that **−0.2** > **−0.3** and **−0.9** < **−0.6**.

EXAMPLE 5.8

Use < or > to order −0.1_−0.8.

Solution

	$-0.1 \quad -0.8$
Write the numbers one under the other, lining up the decimal points.	-0.1 -0.8
They have the same number of digits.	
Since $-1 > -8$, -1 tenth is greater than -8 tenths.	$-0.1 > -0.8$

TRY IT: 5.15 Order each of the following pairs of numbers, using < or >:

−0.3___−0.5

Solution

>

TRY IT: 5.16 Order each of the following pairs of numbers, using < or >:

−0.6___−0.7

Solution

>

Round Decimals

In the United States, gasoline prices are usually written with the decimal part as thousandths of a dollar. For example, a gas station might post the price of unleaded gas at $3.279 per gallon. But if you were to buy exactly one gallon of gas at this price, you would pay $3.28, because the final price would be rounded to the nearest cent. In Whole Numbers, we saw that we round numbers to get an approximate value when the exact value is not needed.

Suppose we wanted to round $2.72 to the nearest dollar. Is it closer to $2 or to $3? What if we wanted to round $2.72 to the nearest ten cents; is it closer to $2.70 or to $2.80? The number lines in Figure 5.4 can help us answer those questions.

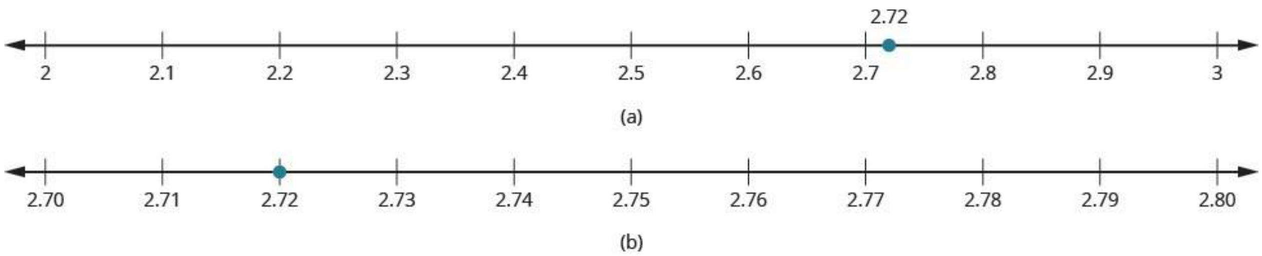


Figure 5.4 ⓐ We see that 2.72 is closer to 3 than to 2. So, 2.72 rounded to the nearest whole number is 3. ⓑ We see that 2.72 is closer to 2.70 than 2.80. So we say that 2.72 rounded to the nearest tenth is 2.7.

Can we round decimals without number lines? Yes! We use a method based on the one we used to round whole numbers.

? HOW TO: ROUND A DECIMAL

Step 1. Locate the given place value and mark it with an arrow.

Step 2. Underline the digit to the right of the given place value.

Step 3. Is this digit greater than or equal to 5?

- Yes - add 1 to the digit in the given place value.
- No - do not change the digit in the given place value

Step 4. Rewrite the number, removing all digits to the right of the given place value.

EXAMPLE 5.9

Round 18.379 to the nearest hundredth.

Solution

	18.379
Locate the hundredths place and mark it with an arrow.	hundredths place ↓ 18.379
Underline the digit to the right of the 7.	hundredths place ↓ 18.379
Because 9 is greater than or equal to 5, add 1 to the 7.	18.379 delete add 1
Rewrite the number, deleting all digits to the right of the hundredths place.	18.38
	18.38 is 18.379 rounded to the nearest hundredth.

TRY IT: 5.17 Round to the nearest hundredth: 1.047.

Solution

1.05

TRY IT: 5.18 Round to the nearest hundredth: 9.173.

Solution

9.17

EXAMPLE 5.10

Round 18.379 to the nearest ⓐ tenth ⓑ whole number.

Solution

ⓐ Round 18.379 to the nearest tenth.

ⓐ Round 18.379 to the nearest tenth.	
	18.379
Locate the tenths place and mark it with an arrow.	tenths place ↓ 18.379
Underline the digit to the right of the tenths digit.	tenths place ↓ $18.3\underline{7}9$
Because 7 is greater than or equal to 5, add 1 to the 3.	$18.3\underbrace{79}_{\text{delete}}$ add 1
Rewrite the number, deleting all digits to the right of the tenths place.	18.4
	So, 18.379 rounded to the nearest tenth is 18.4.
ⓑ Round 18.379 to the nearest whole number.	
	18.379
Locate the ones place and mark it with an arrow.	ones place ↓ 18.379
Underline the digit to the right of the ones place.	ones place ↓ $18.\underline{3}79$
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	$18.\underbrace{379}_{\text{delete}}$ do not add 1
Rewrite the number, deleting all digits to the right of the ones place.	18
	So 18.379 rounded to the nearest whole number is 18.

TRY IT: 5.19 Round 6.582 to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Solution

ⓐ 6.58

ⓑ 6.6

ⓒ 7

TRY IT: 5.20 Round 15.2175 to the nearest ⓐ thousandth ⓑ hundredth ⓒ tenth.

Solution

ⓐ 15.218

ⓑ 15.22

ⓒ 15.2

Review of Key Concepts

- Name a decimal number.
    1. Name the number to the left of the decimal point.
    2. Write "and" for the decimal point.
    3. Name the "number" part to the right of the decimal point as if it were a whole number.
    4. Name the decimal place of the last digit.
- Write a decimal number from its name.
    1. Look for the word "and"—it locates the decimal point.
        a) Place a decimal point under the word "and." Translate the words before "and" into the whole number and place it to the left of the decimal point.
        b) If there is no "and," write a "0" with a decimal point to its right.
    2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
    3. Translate the words after "and" into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
    4. Fill in zeros for place holders as needed.
- Convert a decimal number to a fraction or mixed number.
    1. Look at the number to the left of the decimal.
        a) If it is zero, the decimal converts to a proper fraction.
        b) If it is not zero, the decimal converts to a mixed number. Write the whole number.
    2. Determine the place value of the final digit.

3. Write the fraction. Numerator—the "numbers" to the right of the decimal point; denominator—the place value corresponding to the final digit
4. Simplify the fraction, if possible.

- Order decimals
  1. Check to see if both numbers have the same number of decimal places. If not, write zeros at the end of the one with fewer digits to make them match.
  2. Compare the numbers to the right of the decimal point as if they were whole numbers.
  3. Order the numbers using the appropriate inequality sign.
- Round a decimal
  1. Locate the given place value and mark it with an arrow.
  2. Underline the digit to the right of the given place value.
  3. Is this digit greater than or equal to 5?
     a) Yes - add 1 to the digit in the given place value.
     b) No - do not change the digit in the given place value
- Rewrite the number, removing all digits to the right of the given place value.