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Decimal Operations

Learning Objectives

By the end of this section, you will be able to:

- › Add and subtract decimals
- Multiply decimals
- Divide decimals
- Use decimals in money applications

Add and Subtract Decimals

Let's take one more look at the lunch order from the start of Decimals, this time noticing how the numbers were added together.

All three items (sandwich, water, tax) were priced in dollars and cents, so we lined up the dollars under the dollars and the cents under the cents, with the decimal points lined up between them. Then we just added each column, as if we were adding whole numbers. By lining up decimals this way, we can add or subtract the corresponding place values just as we did with whole numbers.



- Step 1. Write the numbers vertically so the decimal points line up.
- Step 2. Use zeros as place holders, as needed
- Step 3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

Add: 3.7 + 12.4.

Solution

	3.7+12.4
Write the numbers vertically so the decimal points line up.	3.7 +12.4
Place holders are not needed since both numbers have the same number of decimal places.	
Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	1 3.7 +12.4 16.1

> TRY IT: 5.21 Add: 5.7 + 11.9.

⊘ Solution

17.6

> TRY IT: 5.22 Add: 18.32 + 14.79.

Solution

33.11

EXAMPLE 5.12

Add: 23.5 +41.38.

Solution

Write the numbers vertically so the decimal points line up.

Place 0 as a place holder after the 5 in 23.5, so that both numbers have two decimalplaces.	23.5 <mark>0</mark> + 41.38
Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	23.50 + 41.38 64.88

> TRY IT: 5.23 Add: 4.8 + 11.69.

⊘ Solution

16.49

> TRY IT: 5.24 Add: 5.123 + 18.47.

Solution

23.593

How much change would you get if you handed the cashier a \$20 bill for a \$14.65 purchase? We will show the steps to calculate this in the next example.

EXAMPLE 5.13

Subtract: 20 - 14.65.



	20-14.65
Write the numbers vertically so the decimal points line up. Remember 20 is a whole number, so place the decimal point after the 0.	20. 14.65
Place two zeros after the decimal point in 20 as place holders so that both numbers have two decimal places.	20. <mark>00</mark> - 14.65
Subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	2 Ø . Ø Ø - 14.65 5.35

> TRY IT: 5.25

Subtract: 10-9.58.

⊘ Solution

0.42

> **TRY IT**: 5.26

Subtract: 50 -37.42.

Solution

12.58

EXAMPLE 5.14

Subtract: 2.51 - 7.4.

Solution

If we subtract 7.4 from 2.51, the answer will be negative since 7.4 > 2.51. To subtract easily, we can subtract 2.51 from 7.4. Then we will place the negative sign in the result.

	2.51-7.4
Write the numbers vertically so the decimal points line up.	7.4 - 2.51
Place zero after the 4 in 7.4 as a place holder, so that both numbers have two decimal places.	7.4 <mark>0 - 2.51</mark>
Subtract and place the decimal in the answer.	$ \begin{array}{r} 7.40 \\ -2.51 \\ \hline 4.89 \end{array} $
Remember that we are really subtracting 2.51 – 7.4 as a place holder, so that both numbers have two decimal places.	2.51-7.4=-4.89

> TRY IT : 5.27 Subtract: 4.77 - 6.3.

Solution

-1.53

TRY IT : 5.28 Subtract: 8.12 -11.7.

Solution

-3.58

Multiply Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first review multiplying fractions.

Do you remember how to multiply fractions? To multiply fractions, you multiply the numerators and then multiply the denominators.

So let's see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side in Table 5.22. Look for a pattern.

	Α	В
	(0.3)(0.7)	(0.2)(0.46)
Convert to fractions.	$(\frac{3}{10})(\frac{7}{10})$	(2) (46) \(\frac{10}{10}\)
Multiply.	21 100	92 1000
Convert back to decimals.	0.21	0.092

Table 5.22

There is a pattern that we can use. In A, we multiplied two numbers that each had one decimal place, and the product had two decimal places. In B, we multiplied a number with one decimal place by a number with two decimal places, and the product had three decimal places.

How many decimal places would you expect for the product of (0.01)(0.004)? If you said "five," you recognized the pattern. When we multiply two numbers with decimals, we count all the decimal places in the factors—in this case two plus three—to get the number of decimal places in the product—in this case five.

$$(0.01) (0.004) = 0.00004$$
2 places 3 places 5 places
$$\left(\frac{1}{100}\right)\left(\frac{4}{1000}\right) = \frac{4}{100,000}$$

Once we know how to determine the number of digits after the decimal point, we can multiply decimal numbers without converting them to fractions first. The number of decimal places in the product is the sum of the number of decimal places in the factors.

 $The \, rules \, for \, multiplying \, positive \, and \, negative \, numbers \, apply \, to \, decimals, too, of \, course.$

When you multiply signed decimals, first determine the sign of the product and then multiply as if the numbers were both positive. Finally, write the product with the appropriate sign.

HOW TO: MULTIPLY DECIMAL NUMBERS.

- Step 1. Determine the sign of the product.
- Step 2. Write the numbers in vertical format, lining up the numbers on the right.
- Step 3. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
- Step 4. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors. If needed, use zeros as placeholders.
- Step 5. Write the product with the appropriate sign.

Multiply: (3.9)(4.075).



Solution

(3.9)(4.075)

Determine the sign of the product. The signs are the same.	The product will be positive.
Write the numbers in vertical format, lining up the numbers on the right.	4.075 × 3.9
Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.	$ \begin{array}{r} 4.075 \\ \times 3.9 \\ \hline 36675 \\ \underline{12225} \\ 158925 \end{array} $
Place the decimal point. Add the number of decimal places in the factors (1 + 3). Place the decimal point 4 places from the right.	4.075 3 places × 3.9 1 place 36675 12225 158925 4 places
The product is positive.	(3.9)(4.075) = 15.8925

TRY IT: 5.29 Multiply: 4.5(6.107). **⊘** Solution

27.4815

TRY IT: 5.30 Multiply: 10.79(8.12).

Solution

87.6148

EXAMPLE 5.16

Multiply: (-8.2)(5.19).

⊘ Solution

(-8.2)(5.19)The signs are different. The product will be negative. Write in vertical format, lining up the numbers on the right. 5.19 ×8.2 Multiply. 5.19 ×8.2 1038 4152 42558 5.19 ×8.2 Place the decimal point 3 places from the right. 1038 (-8.2)(5.19)4152 1 place 2 places 42.558 (-8.2)(5.19) = -42.558The product is negative.

TRY IT: 5.31 Multiply: (4.63)(-2.9).

Solution

-13.427

TRY IT: 5.32 Multiply: (-7.78)(4.9).

Solution

38.122

In the next example, we'll need to add several placeholder zeros to properly place the decimal point.

Multiply: (0.03)(0.045).



(0.	.03)	(0.0)	45)
ι Ο .	,		

The product is positive.	
Write in vertical format, lining up the numbers on the right.	0.045 × 0.03
Multiply.	$0.045 \times 0.03 = 135$
The decimal point must be 5 places from the right. (0.03)(0.045) 2 places 3 places	0.045 × 0.03 0.00135
Add zeros as needed to get the 5 places.	
The product is positive.	(0.03)(0.045) = 0.00135

> TRY IT: 5.33 Multiply: (0.04)(0.087).

⊘ Solution

0.00348

> TRY IT : 5.34 Multiply: (0.09)(0.067).

Solution

0.00603

Multiply by Powers of 10

In many fields, especially in the sciences, it is common to multiply decimals by powers of 10. Let's see what happens when we multiply 1.9436 by some powers of 10.

1.9436(1000)	1.9436(100)	1.9436(10)
1.9436	1.9436	1.9436
× 1000	× 100	× 10
1943.6000	194 3600	19 4360

Look at the results without the final zeros. Do you notice a pattern?

1.9436(10) = 19.436 1.9436(100) = 194.361.9436(1000) = 1943.6

The number of places that the decimal point moved is the same as the number of zeros in the power of ten. Table 5.26 summarizes the results.

Multiply by	Number of zeros		
10	1	1 place to the right	
100	2	2 places to the right	
1,000	3	3 places to the right	
10,000	4	4 places to the right	

Table 5.26

We can use this pattern as a shortcut to multiply by powers of ten instead of multiplying using the vertical format. We can count the zeros in the power of 10 and then move the decimal point that same of places to the right.

So, for example, to multiply 45.86 by 100, move the decimal point 2 places to the right.

Sometimes when we need to move the decimal point, there are not enough decimal places. In that case, we use zeros as placeholders. For example, let's multiply $2.4\,$ by 100. We need to move the decimal point $2\,$ places to the right. Since there is only one digit to the right of the decimal point, we must write a $0\,$ in the hundredths place.

$$2.4 \times 100 = 240.$$

? HOW TO: MULTIPLY A DECIMAL BY A POWER OF 10.

Step 1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.

Step 2. Write zeros at the end of the number as placeholders if needed.

Multiply 5.63 by factors of (a) 10 (b) 100 (c) 1000.

Solution

By looking at the number of zeros in the multiple of 10, we see the number of places we need to move the decimal to the right.

a	
	56.3(10)
There is 1 zero in 10, so move the decimal point 1 place to the right.	5.63
	56.3

6	
	5.63(100)
There are 2 zeros in 100, so move the decimal point 2 places to the right.	5.63
	563



5.63(1000)

There are 3 zeros in 1000, so move the decimal point 3 places to the right.

A zero must be added at the end.

5.63

5,630

> TRY IT : 5.35

Multiply 2.58 by factors of (a) 10 (b) 100 (c) 1000.

Solution

1. @ 25.8

2. (b) 258

3. © 2,580

> TRY IT : 5.36

Multiply 14.2 by factors of (a) 10 (b) 100 (c) 1000.

Solution

1. @ 142

2. (b) 1,420

3. © 14,200

Divide Decimals

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To understand decimal division, let's consider the multiplication problem

$$(0.2)(4) = 0.8$$

Remember, a multiplication problem can be rephrased as a division problem. So we can write

$$0.8 \div 4 = 0.2$$

We can think of this as "If we divide 8 tenths into four groups, how many are in each group?" Figure 5.6 shows that there are four groups of two-tenths in eight-tenths. So $0.8 \div 4 = 0.2$.

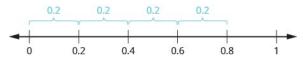


Figure 5.6

Using long division notation, we would write

0.2 4)0.8

Notice that the decimal point in the quotient is directly above the decimal point in the dividend.

To divide a decimal by a whole number, we place the decimal point in the quotient above the decimal point in the dividend and then divide as usual. Sometimes we need to use extra zeros at the end of the dividend to keep dividing until there is no remainder.

? HOW TO: DIVIDE A DECIMAL BY A WHOLE NUMBER.

Step 1. Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.

Step 2. Divide as usual.

EXAMPLE 5.19

Divide: $0.12 \div 3$.



0.12 ÷ 3

Write as long division, placing the decimal point in the quotient above the decimal point

in the dividend.

3)0.12

3)0.12 12 0

Divide as usual. Since 3 does not go into 0 or 1, we use zeros as placeholders.

 $0.12 \div 3 = 0.04$

> TRY IT : 5.37 Divide: 0.28 ÷ 4.

Solution

0.07

> TRY IT : 5.38 Divide: 0.56 ÷ 7.

Solution

0.08

In everyday life, we divide whole numbers into decimals—money—to find the price of one item. For example, suppose a case of 24 water bottles cost \$3.99. To find the price per water bottle, we would divide \$3.99 by 24, and round the answer to the nearest cent (hundredth).

EXAMPLE 5.20

Divide: $$3.99 \div 24$.

Place the decimal point in the quotient above the decimal point in the dividend.

24)3.99

Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth). To do this, we must carry the division to the thousandths place. $3.99 \div 24$

0.166

Round to the nearest cent.

\$0.166 ≈ \$0.17

 $$3.99 \div 24 \approx 0.17

This means the price per bottle is 17 cents.

> **TRY IT**: 5.39

Divide: $$6.99 \div 36$.

Solution

\$0.19



\$0.42

Divide a Decimal by Another Decimal

So far, we have divided a decimal by a whole number. What happens when we divide a decimal by another decimal? Let's look at the same multiplication problem we looked at earlier, but in a different way.

$$(0.2)(4) = 0.8$$

Remember, again, that a multiplication problem can be rephrased as a division problem. This time we ask, "How many times does 0.2 go into 0.8?" Because (0.2)(4) = 0.8, we can say that 0.2 goes into 0.8 four times. This means that

1.8 divided by 0.2 is 4.

We would get the same answer, 4, if we divide 8 by 2, both whole numbers. Why is this so? Let's think about the division problem as a fraction.

 $\begin{array}{r}
 \underbrace{0.8}{0.2} \\
 \underbrace{(0.8)10} \\
 \underbrace{(0.2)10} \\
 \underbrace{8}{2}
 \end{array}$

We multiplied the numerator and denominator by 10 and ended up just dividing 8 by 4. To divide decimals, we multiply both the numerator and denominator by the same power of 10 to make the denominator a whole number. Because of the equivalent fractions property, we haven't changed the value of the fraction. The effect is to move the decimal points in the numerator and denominator the same number of places to the right.

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

It may help to review the vocabulary for division:



PHOW TO: DIVIDE DECIMAL NUMBERS.

- Step 1. Determine the sign of the quotient.
- Step 2. Make the divisor a whole number by moving the decimal point all the way to the right. Move the decimal point in the dividend the same number of places to the right, writing zeros as needed.
- Step 3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
- Step 4. Write the quotient with the appropriate sign.

EXAMPLE 5.21

Divid Sol 2189n+ (3.4).

Solution

Determine the sign of the quotient.

The quotient will be negative.

Make the divisor the whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places to the right.	3.4)2.89
Divide. Place the decimal point in the quotient above the decimal point in the dividend. Add zeros as needed until the remainder is zero.	$ \begin{array}{r} 0.85 \\ 34)28.90 \\ \underline{272} \\ 170 \\ \underline{170} \\ 0 \end{array} $
Write the quotient with the appropriate sign.	$-2.89 \div (3.4) = -0.85$

> TRY IT: 5.41 Divide:

Divide: $-1.989 \div 5.1$.

⊘ Solution

-0.39

> TRY IT : 5.42

Divide: $-2.04 \div 5.1$.

⊘ Solution

-0.4

EXAMPLE 5.22

Divide: $-25.65 \div (-0.06)$.

- 25.	65 -	÷(−	0.	06	3)

The signs are the same.	The quotient is positive.
Make the divisor a whole number by "moving" the decimal point all the way to the right.	0.06)25.65

"Move" the decimal point in the dividend the same number of places.

Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$ \begin{array}{r} 427.5 \\ 006) 2565.0 \\ \underline{-24} \\ 16 \\ \underline{-12} \\ 45 \\ \underline{-42} \\ 30 \\ 30 \end{array} $
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

> TRY IT : 5.43

Divide: $-23.492 \div (-0.04)$.

⊘ Solution

587.3



Divide: $-4.11 \div (-0.12)$.



34.25

Now we will divide a whole number by a decimal number.

Divide: $4 \div 0.05$.

Solution

	4 ÷ 0.05
The signs are the same.	The quotient is
3	positive.
Make the divisor a whole number by "moving" the decimal point all the way to the right. Move the decimal point in the dividend the same number of places, adding zeros as needed.	0.05)4.00
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	80. 5)400. 40 00 00
Write the quotient with the appropriate sign.	4 ÷ 0.05 = 80

We can relate this example to money. How many nickels are there in four dollars? Because $4 \div 0.05 = 80$, there are 80 nickels in \$4.

>

TRY IT: 5.45

Divide: $6 \div 0.03$.

⊘ Solution

200

>

TRY IT: 5.46

Divide: $7 \div 0.02$.

⊘ Solution

350

Use Decimals in Money Applications

We often apply decimals in real life, and most of the applications involving money. The strategy for applications gives us a plan to follow to help find the answer. Take a moment to review that strategy now.

Strategy for Applications

- 1. Identify what you are asked to find.
- 2. Write a phrase that gives the information to find it.
- 3. Translate the phrase to an expression.
- 4. Simplify the expression.
- 5. Answer the question with a complete sentence.

EXAMPLE 5.24

Paul received \$50 for his birthday. He spent \$31.64 on a video game. How much of Paul's birthday money was left?

Solution

What are you asked to find?	How much did Paul have left?
Write a phrase.	\$50 less \$31.64
Translate.	50-31.64
Simplify.	18.36
Write a sentence.	Paul has \$18.36 left.

> TRY IT : 5.47

Nicole earned \$35 for babysitting her cousins, then went to the bookstore and spent \$18.48 on books and coffee. How much of her babysitting money was left?

Solution

\$16.52

> TRY IT : 5.48

Amber bought a pair of shoes for \$24.75 and a purse for \$36.90. The sales tax was \$4.32. How much did Amber spend?

⊘ Solution

\$65.97

Jessie put 8 gallons of gas in her car. One gallon of gas costs \$3.529. How much does Jessie owe for the gas? (Round the answer to the nearest cent.)

Solution

What are you asked to find?	How much did Jessie owe for all the gas?
Write a phrase.	8 times the cost of one gallon of gas
Translate.	8(\$3.529)
Simplify.	\$28.232
Round to the nearest cent.	\$28.23
Write a sentence.	Jessie owes \$28.23 for her gas purchase.

> **TRY IT**: 5.49

Hector put 13 gallons of gas into his car. One gallon of gas costs \$3.175. How much did Hector owe for the gas? Round to the nearest cent.

⊘ Solution

\$41.28

> TRY IT: 5.50

Christopher bought 5 pizzas for the team. Each pizza cost \$9.75. How much did all the pizzas cost?

Solution

\$48.75

Four friends went out for dinner. They shared a large pizza and a pitcher of soda. The total cost of their dinner was \$31.76. If they divide the cost equally, how much should each friend pay?

⊘ Solution

What are you asked to find?	How much should each friend pay?
Write a phrase.	\$31.76 divided equally among the four friends.
Translate to an expression.	\$31.76 ÷4
Simplify.	\$7.94
Write a sentence.	Each friend should pay \$7.94 for his share of the dinner.

> TRY IT : 5.51

Six friends went out for dinner. The total cost of their dinner was \$92.82. If they divide the bill equally, how much should each friend pay?

⊘ Solution

\$15.47

> **TRY IT**: 5.52

Chad worked 40 hours last week and his paycheck was \$570. How much does he earn per hour?

Solution

\$14.25

Be careful to follow the order of operations in the next example. Remember to multiply before you add.

Marla buys 6 bananas that cost \$0.22 each and 4 oranges that cost \$0.49 each. How much is the total cost of the fruit?

Solution

What are you asked to find?	How much is the total cost of the fruit?
Write a phrase.	6 times the cost of each banana plus 4 times the cost of each orange
Translate to an expression.	6(\$0.22) + 4(\$0.49)
Simplify.	\$1.32+\$1.96
Add.	\$3.28
Write a sentence.	Marla's total cost for the fruit is \$3.28.

Stodution_{5.53}

Suzanne buys 3 cans of beans that cost \$0.75 each and 6 cans of corn that cost \$0.62 each. How much is the total cost of these groceries?

Solution

\$5.97



TRY IT: 5.54

Lydia bought movie tickets for the family. She bought two adult tickets for \$9.50 each and four children's tickets for \$6.00 each. How much did the tickets cost Lydia in all?



\$43.00

Key Concepts

- Add or subtract decimals.
 - 1. Write the numbers vertically so the decimal points line up.
 - Use zeros as place holders, as needed.
 - 3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

Multiply decimal numbers.

- 1. Determine the sign of the product.
- 2. Write the numbers in vertical format, lining up the numbers on the right.
- 3. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
- 4. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors. If needed, use zeros as placeholders.
- 5. Write the product with the appropriate sign.

Multiply a decimal by a power of 10.

- 1. Move the decimal point to the right the same number of places as the number of zeros in the power of
- 2. Write zeros at the end of the number as placeholders if needed.

Divide a decimal by a whole number.

- 1. Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.
- 2. Divide as usual.

Divide decimal numbers.

- 1. Determine the sign of the quotient.
- 2. Make the divisor a whole number by moving the decimal point all the way to the right. Move the decimal point in the dividend the same number of places to the right, writing zeros as needed.
- 3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
- 4. Write the quotient with the appropriate sign.

Strategy for Applications

- 1. Identify what you are asked to find.
- 2. Write a phrase that gives the information to find it.
- 3. Translate the phrase to an expression.
- 4. Simplify the expression.
- 5. Answer the question with a complete sentence.