4.4 Add and Subtract Fractions with Common Denominators

Model Fraction Addition

How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that

$$\underset{\text{one quarter}}{\frac{1}{4}} + \underset{\text{two quarters}}{\frac{2}{4}} = \underset{\text{three quarters}}{\frac{3}{4}}$$

Let's use fraction circles to model the same example, $\frac{1}{2} + \frac{2}{4}$

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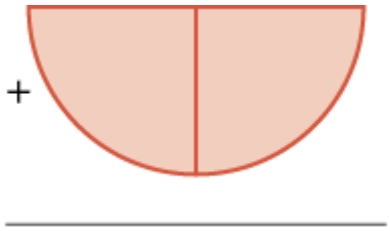
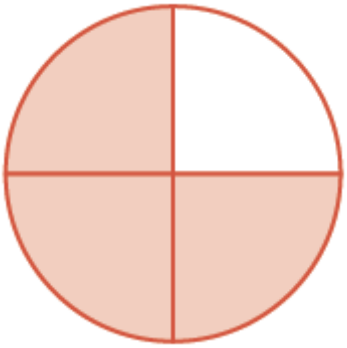
Start with one $\frac{1}{4}$ piece.

Add two more $\frac{1}{4}$ pieces.



The result is $\frac{3}{4}$.



So again, we see that

$$\frac{1}{4}+\frac{2}{4} \equiv \frac{3}{4}$$

NOTE

Doing the Manipulative Mathematics activity Model Fraction Addition will help you develop a better understanding of adding fractions

Example

Exercise

Use a model to find the sum $\frac{3}{8}+\frac{2}{8}$

Solution

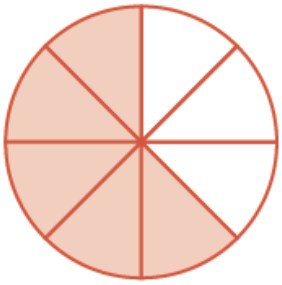
Start with three $\frac{1}{8}$ pieces.

$\frac{5}{8}$

Add two $\frac{1}{8}$ pieces.

$+\ \frac{2}{8}$

How many $\frac{1}{8}$ pieces are there?



$\frac{3}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$.

NOTE

Exercise

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{8}+\frac{4}{8}$$

$\frac{5}{8}$

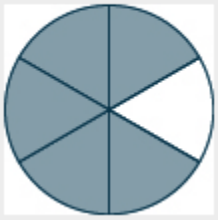
NOTE

Exercise

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{6}+\frac{4}{6}$

$\frac{5}{6}$



Add Fractions with a Common Denominator

The example below shows that to add the same-size pieces—meaning that the fractions have the same **denominator**—we just add the number of pieces.

NOTE: FRACTION ADDITION

If $a, b,$ and c are numbers where $c \neq 0$, then

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

To add fractions with a common denominators, add the numerators and place the sum over the common denominator.

Example

Exercise

Find the sum: $\frac{3}{5}+\frac{1}{5}$.

Solution

	$\frac{3}{5}+\frac{1}{5}$
Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

NOTE

Exercise

Find each sum: $\frac{3}{6}+\frac{2}{6}$

$\frac{5}{6}$

NOTE

Find each sum: $\frac{3}{10}+\frac{7}{10}$.

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NOTE

Exercise

Find the sum: $\frac{x}{3}+\frac{2}{3}$

Solution

	$\frac{x}{3}+\frac{2}{3}$
Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

Note that we cannot simplify this fraction any more. Since x and 2 are not like terms, we cannot combine them.

NOTE

Exercise

Find the sum: $\frac{x}{4}+\frac{3}{4}$.

$\frac{x+3}{4}$

NOTE

Exercise

Find the sum: $\frac{y}{8}+\frac{5}{8}$.

$\frac{y+5}{8}$

Example

Exercise

Find the sum: $-\frac{9}{d}+\frac{3}{d}$.

Solution

We will begin by rewriting the first fraction with the negative sign in the numerator.

$-\frac{a}{b} = \frac{-a}{b}$

	$-\frac{9}{d} + \frac{3}{d}$
Rewrite the first fraction with the negative in the numerator.	$\frac{-9}{d} + \frac{9}{d}$
Add the numerators and place the sum over the common denominator.	$\frac{-9+3}{d}$
Simplify the numerator.	$\frac{-6}{d}$
Rewrite with negative sign in front of the fraction.	$-\frac{6}{d}$

NOTE

Exercise

Find the sum: $-\frac{7}{d} + \frac{8}{d}$

$\frac{1}{d}$

NOTE

Exercise

Find the sum: $-\frac{6}{m} + \frac{9}{m}$.

$\frac{3}{m}$

EXAMPLE

Exercise

Find the sum: $\frac{2n}{11} + \frac{5n}{11}$.

Solution

	$\frac{2n}{11}+\frac{5n}{11}$
Add the numerators and place the sum over the common denominator.	$\frac{2n+5n}{11}$
Combine like terms.	$\frac{7n}{11}$

NOTE

Exercise

Find the sum: $\frac{3p}{8}+\frac{6p}{8}$.

$\frac{9p}{8}$

NOTE

Exercise

Find the sum: $\frac{2q}{5}+\frac{7q}{5}$.

$\frac{9q}{5}$

Example

Exercise

Find the sum: $-\frac{3}{12}+\left(-\frac{5}{12}\right)$

Solution

	$-\frac{3}{12}+\left(-\frac{5}{12}\right)$
Add the numerators and place the sum over the common denominator.	$\frac{-3+(-5)}{12}$
Add.	$\frac{-8}{12}$
Simplify the fraction.	$-\frac{2}{3}$

Find each sum: $-\frac{4}{15}+\left(-\frac{6}{15}\right)$

$-\frac{2}{3}$

NOTE

Exercise

Find each sum: $-\frac{5}{21}+\left(-\frac{9}{21}\right)$.

$-\frac{2}{3}$

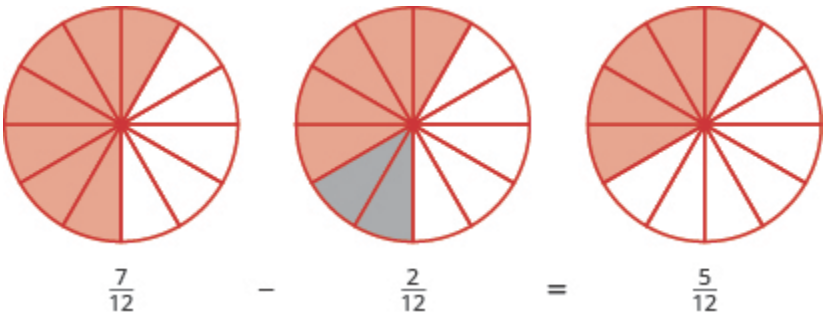
Model Fraction Subtraction

Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

$$\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12} - \frac{2}{12}$.

Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?



Again, we have five twelfths, $\frac{5}{12}$.

Doing the Manipulative Mathematics activity Model Fraction Subtraction will help you develop a better understanding of subtracting fractions.

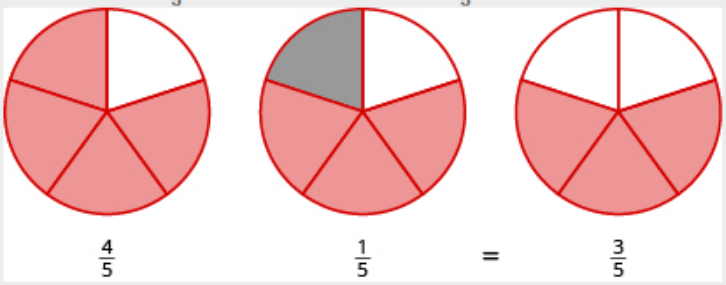
Use fraction circles to find the difference: $\frac{4}{5} - \frac{1}{5}$.

Example

Exercise

Solution

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



NOTE

Exercise

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{7}{8}-\frac{4}{8}$

NOTE

Exercise

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{5}{6}-\frac{4}{6}$

Subtract Fractions with a Common Denominator

We subtract fractions with a common **denominator** in much the same way as we add fractions with a common denominator.

NOTE: FRACTION SUBTRACTION

If $a, b,$ and c are numbers where $c \neq 0$, then

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

To subtract fractions with a common denominators, we subtract the numerators and place the difference over the common denominator.

Example

Exercise

Find the difference: $\frac{23}{24} - \frac{14}{24}$.

Solution

	$\frac{23}{24} - \frac{14}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

NOTE

Exercise

Find the difference: $\frac{19}{28} - \frac{7}{28}$.

$\frac{3}{7}$

NOTE

Exercise

Find the difference: $\frac{27}{32} - \frac{11}{32}$.

$\frac{1}{2}$

Example

Exercise

Find the difference: $\frac{y}{6} - \frac{1}{6}$.

Solution

	$\frac{y}{6} - \frac{1}{6}$
Subtract the numerators and place the difference over the common denominator.	$\frac{y-1}{6}$

The fraction is simplified because we cannot combine the terms in the numerator.

	$\frac{y}{6} - \frac{1}{6}$
Subtract the numerators and place the difference over the common denominator.	$\frac{y-1}{6}$

NOTE

Exercise

Find the difference: $\frac{x}{7} - \frac{2}{7}$.

$\frac{x-2}{7}$

NOTE

Exercise

Find the difference: $\frac{y}{14} - \frac{13}{14}$.

$\frac{y-13}{14}$

Example

Exercise

Find the difference: $-\frac{10}{x}-\frac{4}{x}$.

Solution

Remember, the fraction $-\frac{10}{x}$ can be written as $\frac{-10}{x}$.

	$-\frac{10}{x}-\frac{4}{x}$
Subtract the numerators.	$\frac{-10-4}{x}$
Simplify.	$\frac{-14}{x}$
Rewrite with the negative sign in front of the fraction.	$-\frac{14}{x}$

NOTE

Exercise

Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

$-\frac{16}{x}$

NOTE

Exercise

Find the difference: $-\frac{17}{a}-\frac{5}{a}$

$$-\frac{22}{a}$$

Now let's do an example that involves both addition and subtraction.

Example

Exercise

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Solution

	$\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$
Combine the numerators over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify the numerator, working left to right.	$\frac{-2-1}{8}$
Subtract the terms in the numerator.	$\frac{-3}{8}$
Rewrite with the negative sign in front of the fraction.	$-\frac{3}{8}$

NOTE

Exercise

Simplify: $\frac{2}{5}+\left(-\frac{4}{5}\right)-\frac{3}{5}$.

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NOTE

Exercise

Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

$-\frac{2}{3}$

Key Concepts

- **Fraction Addition**
  - If a, b and c are numbers where $c \neq 0$, then $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$
  - To add fractions, add the numerators and place the sum over the common denominator.
- **Fraction Subtraction**
  - If $a, b,$ and c are numbers where $c \neq 0$, then $\frac{a}{c}.-\frac{b}{c}=\frac{a-b}{c}$
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.