4.5 Add and Subtract Fractions with Different Denominators

Learning Objectives

By the end of this section, you will be able to:

- Find the least common denominator (LCD)
- Convert fractions to equivalent fractions with the LCD
- Add and subtract fractions with different denominators
- Identify and use fraction operations
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Find the Least Common Denominator

In the previous section, we explained how to add and subtract fractions with a common denominator. But how can we add and subtract fractions with unlike denominators?

Let's think about coins again. Can you add one quarter and one dime? You could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit—cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents. See Figure 4.7.

Figure 4.7 **Together, a quarter and a dime are worth 35 cents, or $\frac{35}{100}$ a dollar.**

Similarly, when we add fractions with different denominators, we have to convert them to equivalent fractions with a common denominator. With the coins, when we convert to cents, the denominator is 100. Since there are 100 cents in one dollar, 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$. So we add $\frac{25}{100} + \frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

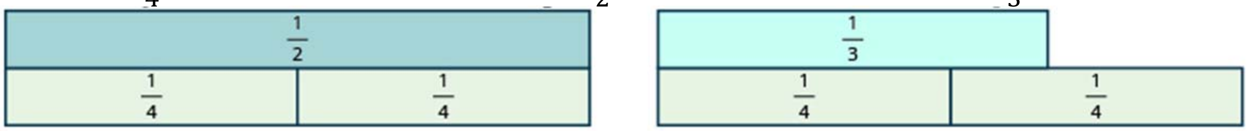
You practiced adding and subtracting fractions with common denominators. Now let's see what you need to do with fractions that have different denominators.

First, we will use fraction tiles to model finding the common denominator of $\frac{1}{2}$ and $\frac{1}{3}$.

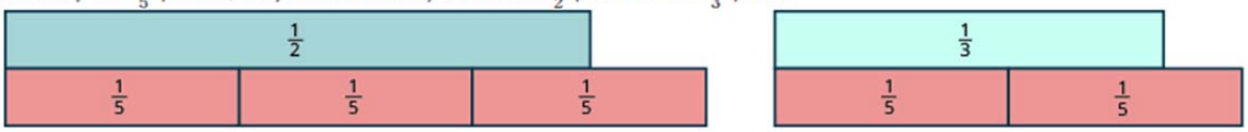
We'll start with one $\frac{1}{2}$ tile and $\frac{1}{3}$ tile. We want to find a common fractions that we can use to match both $\frac{1}{2}$

and $\frac{1}{3}$ exactly.

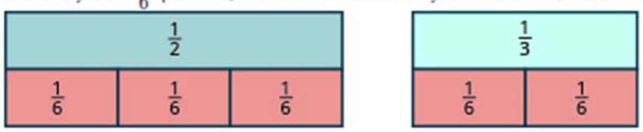
If we try the $\frac{1}{4}$ pieces, 2 of them exactly match the $\frac{1}{2}$ piece, but they do not match the $\frac{1}{3}$ piece.



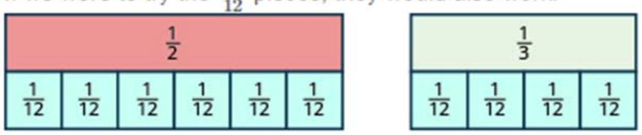
If we try the $\frac{1}{5}$ pieces, they do not exactly cover the $\frac{1}{2}$ piece or the $\frac{1}{3}$ piece.



If we try the $\frac{1}{6}$ pieces, we see that exactly 3 of them cover the $\frac{1}{2}$ piece, and exactly 2 of them cover the $\frac{1}{3}$ piece.



If we were to try the $\frac{1}{12}$ pieces, they would also work.



Even smaller tiles, such as $\frac{1}{24}$ and $\frac{1}{48}$, would also exactly cover the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece.

The denominator of the largest piece that covers both fractions is the **least common denominator (LCD)** of the two fractions. So, the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Notice that all of the tiles that cover $\frac{1}{2}$ and $\frac{1}{3}$ have something in common: Their denominators are common multiples of 2 and 3, the denominators of $\frac{1}{2}$ and $\frac{1}{3}$. The least common multiple (LCM) of the denominators is 6, and so we say that 6 is the least common denominator (LCD) of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

NOTE
Doing the Manipulative Mathematics activity Finding the Least Common Denominator will help you develop a better understanding of the LCD.

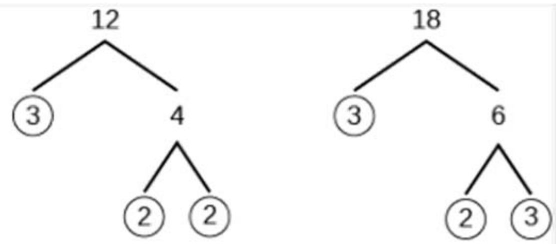
NOTE: Least Common Denominator
The **least common denominator (LCD)** of two fractions is the least common multiple (LCM) of their denominators.

To find the LCD of two fractions, we will find the LCM of their denominators. We follow the procedure we used earlier to find the LCM of two numbers. We only use the denominators of the fractions, not the numerators, when finding the LCD.

EXAMPLE 4.63

Find the LCD for the fractions $\frac{7}{12}$ and $\frac{5}{18}$.

Factor each denominator into its primes.



List the primes of 12 and the primes of 18 lining them up in columns when possible.

$$\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 \\ 18 &= 2 \cdot \quad\;\; 3 \cdot 3 \end{aligned}$$

Bring down the columns.

$$\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 \\ 18 &= 2 \cdot \quad\;\; 3 \cdot 3 \\ \text{LCM} &= 2 \cdot 2 \cdot 3 \cdot 3 \end{aligned}$$

Multiply the factors. The product is the LCM.

$\text{LCM} = 36$

The LCM of 12 and 18 is 36, so the LCD of $\frac{7}{12}$ and $\frac{5}{18}$ is 36.

LCD of $\frac{7}{12}$ and $\frac{5}{18}$ is 36.

TRY IT: 4.125

Find the least common denominator for the fractions: $\frac{7}{12}$ and $\frac{11}{15}$.

Solution

60

TRY IT: 4.126

Find the least common denominator for the fractions: $\frac{13}{15}$ and $\frac{17}{5}$.

Solution

15

To find the LCD of two fractions, find the LCM of their denominators. Notice how the steps shown below are similar to the steps we took to find the LCM.

HOW TO: FIND THE LEAST COMMON DENOMINATOR (LCD) OF TWO FRACTIONS.

Step 1. Factor each denominator into its primes.

Step 2. List the primes, matching primes in columns when possible.

Step 3. Bring down the columns.

Step 4. Multiply the factors. The product is the LCM of the denominators.

Step 5. The LCM of the denominators is the LCD of the fractions.

EXAMPLE 4.64

Find the least common denominator for the fractions $\frac{8}{15}$ and $\frac{11}{24}$

Solution

To find the LCD, we find the LCM of the denominators.

Find the LCM of **15** and **24.**

$$\begin{array}{l} 15 = \qquad\quad\;\; 3 \cdot 5 \\ 24 = 2 \cdot 2 \cdot 2 \cdot 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\ \text{LCD} = 120 \end{array}$$

The LCM of **15** and 24 is **120**. So, the LCD of $\frac{8}{15}$ and $\frac{11}{24}$ is **120.**

TRY IT: 4.127

Find the least common denominator for the fractions: $\frac{13}{24}$ and $\frac{17}{32}$

Solution

96

TRY IT: 4.128

Find the least common denominator for the fractions: $\frac{9}{28}$ and $\frac{21}{32}$

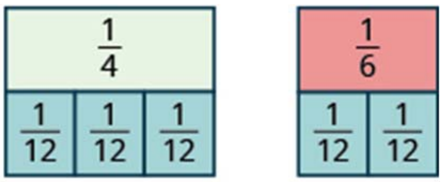
Solution

224

Convert Fractions to Equivalent Fractions with the LCD

Earlier, we used fraction tiles to see that the LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12. We saw that three $\frac{1}{12}$ pieces exactly covered $\frac{1}{4}$ and two $\frac{1}{12}$ pieces exactly covered $\frac{1}{6}$, so

$$\frac{1}{4} = \frac{3}{12} \ and \ \frac{1}{6} = \frac{2}{12}.$$



We say that $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent fractions and also that $\frac{1}{6}$ and $\frac{2}{12}$ are equivalent fractions.

We can use the equivalent fractions property to algebraically change a fraction to an equivalent one. Remember, two fractions are equivalent if they have the same value. The equivalent fractions property is repeated below for reference.

EQUIVALENT FRACTIONS PROPERTY

If ***a, b, c*** are whole numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \ and \ \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

To add or subtract fractions with different denominators, we will first have to convert each fraction to an **equivalent fraction** with the LCD. Let's see how to change $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12 without using models.

EXAMPLE 4.65

Convert to $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12, their LCD.

Solution

Find the LCD.	The LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12.
Find the number to multiply 4 to get 12.	$4 \cdot 3 = 12$
Find the number to multiply 6 to get 12.	$6 \cdot 2 = 12$
Use the equivalent fractions property to convert each fraction to an equivalent fraction with the LCD, multiplying both the numerator and denominator of each fraction by the same number.	$\frac{1}{4}$ $\frac{1}{6}$ $\frac{1 \cdot 3}{4 \cdot 3}$ $\frac{1-2}{6-2}$
Simplify the numerators and denominators	$\frac{3}{12}$ $\frac{2}{12}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{1}{6} = \frac{2}{12}$
We do not reduce the resulting fractions. If we did, we would get back to our original fractions and lose the common denominator.	

TRY IT: 4.129

Change to equivalent fractions with the LCD:

$\frac{3}{4}$ and $\frac{5}{6}$, LCD = 12

Solution

$\frac{9}{12}, \frac{10}{12}$

TRY IT: 4.130

Change to equivalent fractions with the LCD:

$-\frac{7}{12}$ and $\frac{11}{15}$, LCD = 60

Solution

$-\frac{35}{60}, \frac{44}{60}$

HOW TO: CONVERT TWO FRACTIONS TO EQUIVALENT FRACTIONS WITH THEIR LCD AS THE COMMON DENOMINATOR.

Step 1. Find the LCD

Step 2. For each fraction, determine the number needed to multiply the denominator to get the LCD

Step 3. Use the equivalent fractions property to multiply both the numerator and denominator by the number you found in Step 2.

Step 4. Simplify the numerator and denominator.

EXAMPLE 4.66

Convert $\frac{8}{15}$ and $\frac{11}{24}$ to equivalent fractions with denominator 120, their LCD.

Solution

	The LCD is 120. We will start at Step 2.
Find the number that must multiply 15 to get 120.	$15 \cdot 8 = 120$
Find the number that must multiply 24 to get 120.	$24 \cdot 5 = 120$
Use the Equivalent Fractions Property.	$\frac{8 \cdot 8}{15 \cdot 8}$ $\frac{11 \cdot 5}{24 \cdot 5}$
Simplify the numerators and denominators.	$\frac{64}{120}$ $\frac{55}{120}$

TRY IT: 4.131 Change to equivalent fractions with the LCD: $\frac{13}{24}$ and $\frac{17}{32}$, LCD 96

Solution

$\frac{52}{96}, \frac{51}{96}$

TRY IT: 4.132 Change to equivalent fractions with the LCD: $\frac{9}{28}$ and $\frac{27}{32}$, LCD 224

Solution

$\frac{72}{224}, \frac{189}{224}$

Add and Subtract Fractions with Different Denominators

Once we have converted two fractions to equivalent forms with common denominators, we can add or subtract them by adding or subtracting the numerators.

HOW TO: ADD OR SUBTRACT FRACTIONS WITH DIFFERENT DENOMINATORS.

Step 1. Find the LCD.

Step 2. Convert each fraction to an equivalent form with the LCD as the denominator.

Step 3. Add or subtract the fractions.

Step 4 . Write the result in simplified form.

EXAMPLE 4.67

Add $\frac{1}{2}$ and $\frac{1}{3}$

Solution

	$\frac{1}{2}+\frac{1}{3}$
Find the LCD of 2, 3. $2 = 2$ $3 = \quad 3$ $\text{LCD} = 2 \cdot 3$ $\text{LCD} = 6$	
Change into equivalent fractions with the LCD 6.	$\frac{1 \cdot 3}{2 \cdot 3}+\frac{1 \cdot 2}{3 \cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{6}+\frac{2}{6}$
Add.	$\frac{5}{6}$

Remember, always check to see if the answer can be simplified. Since **5** and **6** have no common factors, the fraction $\frac{5}{6}$ cannot be reduced.

TRY IT: 4.133 Add: $\frac{1}{4}+\frac{1}{3}$.

Solution

$\frac{7}{12}$

TRY IT: 4.134 Add: $\frac{1}{2}+\frac{1}{5}$.

Solution

$\frac{7}{10}$

EXAMPLE 4.68

Subtract: $\frac{1}{2}-\left(-\frac{1}{4}\right)$.

Solution

	$\frac{1}{2}-\left(-\frac{1}{4}\right)$
Find the LCD of 2 and 4. $\begin{array}{r} 2=2 \\ 4=2\cdot 2 \\ \hline \text{LCD}=2\cdot 2 \\ \text{LCD}=4 \end{array}$	
Rewrite as equivalent fractions using the LCD 4.	$\frac{1\cdot 2}{2\cdot 2}-\left(-\frac{1}{4}\right)$
Simplify the first fraction.	$\frac{2}{4}-\left(-\frac{1}{4}\right)$
Subtract.	$\frac{2-(-1)}{4}$
Simplify.	$\frac{3}{4}$

One of the fractions already had the least common denominator, so we only had to convert the other fraction.

TRY IT: 4.135 Simplify: $\frac{1}{2}-\left(-\frac{1}{8}\right)$.

Solution

$\frac{5}{8}$

TRY IT: 4.136 Simplify: $\frac{1}{3} - \left(-\frac{1}{6}\right)$.

Solution

$\frac{1}{2}$

EXAMPLE 4.69

Add: $\frac{7}{12} + \frac{5}{18}$

Solution

$\frac{7}{12} + \frac{5}{18}$	
Find the LCD of 12 and 18. $$\begin{array}{l} 12 = 2 \cdot 2 \cdot 3 \\ 18 = 2 \cdot \quad 3 \cdot 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCD} = 36 \end{array}$$	
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 3}{12 \cdot 3} + \frac{5 \cdot 2}{18 \cdot 2}$
Simplify the numerators and denominators.	$\frac{21}{36} + \frac{10}{36}$
Add.	$\frac{31}{36}$

Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.

TRY IT: 4.138

Add: $\frac{7}{12} + \frac{11}{15}$

Solution

$\frac{79}{60}$

TRY IT: 4.139

Add: $\frac{13}{15} + \frac{17}{20}$

Solution

$\frac{103}{60}$

When we use the equivalent fractions property, there is a quick way to find the number you need to multiply by to get the LCD. Write the factors of the denominators and the LCD just as you did to find the LCD. The “missing” factors of each denominator are the numbers you need.

missing factors

$12 = 2 \cdot 2 \cdot 3$

$18 = 2 \cdot \quad 3 \cdot 3$

$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3$

$\text{LCD} = 36$

The LCD, **36**, has **2** factors of **2** and **2** factors of **3**.

Twelve has two factors of **2**, but only one of **3** —so it is "missing"one **3**. We multiplied the numerator and denominator of $\frac{7}{12}$ by **3** to get an equivalent fraction with denominator **36**.

Eighteen is missing one factor of 2 —so you multiply the numerator and denominator $\frac{\mathbf{5}}{\mathbf{18}}$ by **2** to get an equivalent fraction with denominator **36**. We will apply this method as we subtract the fractions in the next example.

EXAMPLE 4.70

Subtract: $\frac{7}{15} - \frac{19}{24}$

Solution

	$\frac{7}{15} - \frac{19}{24}$
Find the LCD. $15 = 3 \cdot 5$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ $\text{LCD} = 120$ 15 is "missing" three factors of 2 24 is "missing" a factor of 5	
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 8}{15 \cdot 8} - \frac{19 \cdot 5}{24 \cdot 5}$
Simplify each numerator and denominator.	$\frac{56}{120} - \frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Rewrite showing the common factor of 3.	$-\frac{13.3}{40.3}$
Remove the common factor to simplify.	$-\frac{13}{40}$

TRY IT: 4.139 Subtract: $\frac{13}{24} - \frac{17}{32}$

Solution

$\frac{1}{96}$

TRY IT: 4.140 Subtract: $\frac{21}{32} - \frac{9}{28}$

Solution

$\frac{75}{224}$

EXAMPLE 4.71

Add: $-\frac{11}{30}+\frac{23}{42}$

Solution

	$-\frac{11}{30}+\frac{23}{42}$
Find the LCD. $30 = 2\cdot3\cdot5$ $42 = 2\cdot3\cdot\quad 7$ $\text{LCD} = 2\cdot3\cdot5\cdot7$ $\text{LCD} = 210$	
Rewrite as equivalent fractions with the LCD	$-\frac{11\cdot7}{30\cdot7}-\frac{23\cdot5}{42\cdot5}$
Simplify each numerator and denominator.	$-\frac{77}{210}+\frac{115}{210}$
Add.	$\frac{38}{210}$
Rewrite showing the common factor of 2.	$\frac{19\cdot2}{105\cdot2}$
Remove the common factor to simplify.	$\frac{19}{105}$

TRY IT: 4.141 Add: $-\frac{13}{42}+\frac{17}{35}$.

Solution

$\frac{37}{210}$

TRY IT: 4.142 Add: $-\frac{19}{24}+\frac{17}{32}$.

Solution

$-\frac{25}{96}$

In the next example, one of the fractions has a variable in its numerator. We follow the same steps as when both numerators are numbers.

EXAMPLE 4.72

Add: $\frac{3}{5}+\frac{x}{8}$.

Solution

The fractions have different denominators.

	$\frac{3}{5}+\frac{x}{8}$
Find the LCD. $\begin{array}{r} 5 = \qquad\quad 5 \\ 8 = 2 \cdot 2 \cdot 2 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 5 \\ \text{LCD} = 40 \end{array}$	
Rewrite as equivalent fractions with the LCD.	$\frac{3 \cdot 8}{5 \cdot 8}+\frac{x \cdot 5}{8 \cdot 5}$
Simplify the numerators and denominators.	$\frac{24}{40}+\frac{5x}{8}$
Add.	$\frac{24+5x}{40}$

We cannot add **24** and **5*x*** since they are not like terms, so we cannot simplify the expression any further.

TRY IT: 4.143 Add: $\frac{y}{6}+\frac{7}{9}$.

Solution

$\frac{3y+14}{18}$

TRY IT: 4.144 Add: $\frac{x}{6}+\frac{7}{15}$.

Solution

$\frac{5x+14}{30}$

Identify and Use Fraction Operations

By now in this chapter, you have practiced multiplying, dividing, adding, and subtracting fractions. The following table summarizes these four fraction operations. Remember: You need a common denominator to add or subtract fractions, but not to multiply or divide fractions

Summary of Fraction Operations

Fraction multiplication: Multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Fraction division: Multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Fraction addition: Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Fraction subtraction: Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Simplify:

1. ⓐ$-\frac{1}{4} + \frac{1}{6}$
2. ⓑ$-\frac{1}{4} \div \frac{1}{6}$

Solution

First we ask ourselves, “What is the operation?”

ⒶThe operation is addition.

Do the fractions have a common denominator? No.

	$-\frac{1}{4}+\frac{1}{6}$
Find the LCD. $$\begin{aligned} 4 &= 2\cdot 2 \\ 6 &= 2\cdot \quad 3 \\ \hline \text{LCD} &= 2\cdot 2\cdot 3 \\ \text{LCD} &= 12 \end{aligned}$$	
Rewrite each fraction as an equivalent fraction with the LCD.	$-\frac{1\cdot 3}{4\cdot 3}+\frac{1\cdot 2}{6\cdot 2}$
Simplify the numerators and denominators.	$-\frac{3}{12}+\frac{2}{12}$
Add the numerators and place the sum over the common denominator.	$-\frac{1}{12}$
Check to see if the answer can be simplified. It cannot.	

ⓑThe operation is division. We do not need a common denominator.

	$-\frac{1}{4}\div\frac{1}{6}$
To divide fractions, multiply the first fraction by the reciprocal of the second.	$-\frac{1}{4}\cdot\frac{6}{1}$
Multiply.	$-\frac{6}{4}$
Simplify.	$-\frac{3}{2}$

TRY IT: 4.145 Simplify each expression:

1. ⓐ$-\frac{3}{4}-\frac{1}{6}$
2. ⓑ$-\frac{3}{4}\cdot\frac{1}{6}$

Solution

1. ⓐ$-\frac{11}{12}$
2. ⓑ$-\frac{1}{8}$

TRY IT: 4.146 Simplify each expression:

1. ⓐ$\frac{5}{6}\div\left(-\frac{1}{4}\right)$
2. ⓑ$\frac{5}{6}-\left(-\frac{1}{4}\right)$

Solution

1. ⓐ$-\frac{10}{3}$
2. ⓑ$-\frac{13}{12}$

EXAMPLE 4.74

Simplify:

1. ⓐ $\frac{5}{x} - \frac{3}{10}$
2. ⓑ $\frac{5}{x} \cdot \frac{3}{10}$

Solution

ⓐ The operation is subtraction. The fractions do not have a common denominator.

	$\frac{5x}{6} - \frac{3}{10}$
Rewrite each fraction as an equivalent fraction with the LCD, 30.	$\frac{5x \cdot 5}{6 \cdot 5} - \frac{3 \cdot 3}{10 \cdot 3}$
	$\frac{25x}{30} - \frac{9}{30}$
Subtract the numerators and place the difference over the common denominator.	$\frac{25x - 9}{30}$
ⓑ The operation is multiplication; no need for a common denominator.	
	$\frac{5x}{6} \cdot \frac{3}{10}$
To multiply fractions, multiply the numerators and multiply the denominators.	$\frac{5x \cdot 3}{6 \cdot 10}$
Rewrite, showing common factors.	$\frac{\cancel{5} \cdot x \cdot \cancel{3}}{2 \cdot \cancel{3} \cdot 2 \cdot \cancel{5}}$
Remove common factors to simplify.	$\frac{x}{4}$

TRY IT: 4.147 Simplify:

1. ⓐ $\frac{3}{a} - \frac{8}{9}$
2. ⓑ $\frac{3}{a} \cdot \frac{8}{9}$

Solution

1. ⓐ $\frac{27}{a}$
2. ⓑ $\frac{2}{a}$

TRY IT: 4.148 Simplify:

1. ⓐ $\frac{4}{k} + \frac{5}{6}$
2. ⓑ $\frac{4}{k} \div \frac{5}{6}$

Solution

1. ⓐ $\frac{24}{k}$
2. ⓑ $\frac{24}{k}$

Use the Order of Operations to Simplify Complex Fractions

In Multiply and Divide Mixed Numbers and Complex Fractions, we saw that a complex fraction is a fraction in which the numerator or denominator contains a fraction. We simplified complex fractions by rewriting them as division problems. For example,

$$\frac{\frac{3}{4}}{\frac{5}{8}} = \frac{3}{4} \div \frac{5}{8}$$

Now we will look at complex fractions in which the numerator or denominator can be simplified. To follow the order of operations, we simplify the numerator and denominator separately first. Then we divide the numerator by the denominator.

HOW TO: SIMPLIFY COMPLEX FRACTIONS.

Step 1. Simplify the numerator.

Step 2. Simplify the denominator.

Step 3. Divide the numerator by the denominator.

Step 4. Simplify if possible.

EXAMPLE 4.75

Simplify: $\dfrac{\left(\frac{1}{2}\right)^2}{4+3^2}$

	$\dfrac{\left(\frac{1}{2}\right)^2}{4+3^2}$
Simplify the numerator.	$\dfrac{\frac{1}{4}}{4+3^2}$
Simplify the term with the exponent in the denominator.	$\dfrac{\frac{1}{4}}{4+9}$
Add the terms in the denominator.	$\dfrac{\frac{1}{4}}{13}$
Divide the numerator by the denominator.	$\frac{1}{4} \div 13$
Rewrite as multiplication by the reciprocal.	$\frac{1}{4} \cdot \frac{1}{13}$
Multiply.	$\frac{1}{52}$

TRY IT: 4.149 Simplify: $\dfrac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Solution

$\dfrac{1}{90}$

TRY IT: 4.150 Simplify: $\dfrac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Solution

272

EXAMPLE 4.76

Simplify: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution

	$\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$
Rewrite numerator with the LCD of 6 and denominator with LCD of 12.	$\dfrac{\frac{3}{6}+\frac{4}{6}}{\frac{9}{12}-\frac{2}{12}}$
Add in the numerator. Subtract in the denominator.	$\dfrac{\frac{7}{6}}{\frac{7}{12}}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Rewrite as multiplication by the reciprocal.	$\frac{7}{6} \cdot \frac{12}{7}$
Rewrite, showing common factors.	$\dfrac{\cancel{7} \cdot \cancel{6} \cdot 2}{\cancel{6} \cdot \cancel{7} \cdot 1}$
Simplify.	2

TRY IT: **4.151** Simplify: $\dfrac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$

Solution

2

TRY IT: 4.152 Simplify: $\dfrac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$

Solution

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can also evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

EXAMPLE 4.77

Evaluate $x+\frac{1}{3}$ when

1. ⓐ $x=-\frac{1}{3}$
2. ⓑ $x=-\frac{3}{4}$.

Solution

ⓐ To evaluate $x+\frac{1}{3}$ when $x=-\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

ⓑ To evaluate $x+\frac{1}{3}$ when $x=-\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{3}{4}$ for x.	$-\frac{3}{4}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot3}{4\cdot3}+\frac{1\cdot4}{3\cdot4}$
Simplify the numerators and denominators.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

TRY IT: 4.153 Evaluate $x + \frac{3}{4}$ when

1. ⓐ $x = -\frac{7}{4}$
2. ⓑ $x = -\frac{5}{4}$

Solution

1. ⓐ-1
2. ⓑ$-\frac{1}{2}$

TRY IT: 4.154 Evaluate: $y + \frac{1}{2}$ when

1. ⓐ $y = \frac{2}{3}$
2. ⓑ $y = -\frac{3}{4}$

Solution

1. ⓐ $\frac{7}{6}$
2. ⓑ$-\frac{1}{4}$

EXAMPLE 4.78

Evaluate: $y - \frac{5}{6}$ when $y = -\frac{2}{3}$.

Solution

We substitute $-\frac{2}{3}$ for y in the expression.

	$y - \frac{5}{6}$
Substitute $-\frac{2}{3}$ for y.	$-\frac{2}{3} - \frac{5}{6}$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{4}{6} - \frac{5}{6}$
Subtract.	$-\frac{9}{6}$
Simplify.	$-\frac{3}{2}$

TRY IT: 4.155 Evaluate: $y - \frac{1}{2}$ when $y = -\frac{1}{4}$.

Solution

$-\frac{3}{4}$

TRY IT: 4.156 Evaluate: $x - \frac{3}{8}$ when $x = -\frac{5}{2}$.

Solution

$-\frac{23}{8}$

EXAMPLE 4.79

Evaluate: $2x^2y$ when $x = \frac{1}{4}$ *and* $y = -\frac{2}{3}$.

Solution

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. The product will be negative.	$-\frac{2}{1}\cdot\frac{1}{16}\cdot\frac{2}{3}$
Simplify.	$-\frac{4}{48}$
Remove the common factors.	$-\frac{1\cdot\cancel{4}}{\cancel{4}\cdot 12}$
Simplify.	$-\frac{1}{12}$

TRY IT: 4.157 Evaluate: $3ab^2$ when $a = -\frac{2}{3}$ *and* $b = -\frac{1}{2}$.

Solution

$-\frac{1}{2}$

TRY IT: 4.158 Evaluate: $4c^2d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$.

Solution

$\frac{2}{3}$

EXAMPLE 4.80

Evaluate: $\frac{p+q}{r}$ when $p = -4, q = -2, and\ r = 8.$

Solution

We substitute the values into the expression and simplify.

	$\frac{p+q}{r}$
Substitute −4 for *p*, −2 for *q* and 8 for *r*.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$-\frac{6}{8}$
Simplify.	$-\frac{3}{4}$

TRY IT: 4.159 Evaluate: $\frac{a+b}{c}$ when $a = -8, b = -7, and\ c = 6.$

Solution

$-\frac{5}{2}$

TRY IT: 4.160 Evaluate: $\frac{x+y}{x}$ when $x = 9,\ y = -18, and\ z = -6.$

Solution

$\frac{3}{2}$

Key Concepts

- **Find the least common denominator (LCD) of two fractions.**
    1. Factor each denominator into its primes.
    2. List the primes, matching primes in columns when possible.
    3. Bring down the columns.
    4. Multiply the factors. The product is the LCM of the denominators.
    5. The LCM of the denominators is the LCD of the fractions.
- Equivalent Fractions Property
    - If ***a, b, c*** are whole numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \; and \; \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

- **Convert two fractions to equivalent fractions with their LCD as the common denominator.**
    1. Find the LCD.
    2. For each fraction, determine the number needed to multiply the denominator to get the LCD.
    3. Use the equivalent fractions property to multiply the numerator and denominator by the number from Step 2.
    4. Simplify the numerator and denominator.
- **Add or subtract fractions with different denominators.**
    1. Find the LCD.
    2. Convert each fraction to an equivalent form with the LCD as the denominator.
    3. Add or subtract the fractions.
    4. Write the result in simplified form.
- Summary of Fraction Operations
    - **Fraction multiplication:** Multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

    - **Fraction division:** Multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

- **Fraction addition:** Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

- **Fraction subtraction:** Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

- **Simplify complex fractions.**
  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Divide the numerator by the denominator.
  4. Simplify if possible.